Eclipse in Occlusion
A perspectival mereotopological representation of celestial eclipses

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Abstract

This paper presents a formalisation and exploration of the concept of *eclipse* from the perspective of qualitative spatial reasoning. Building upon theories of spatial connectivity, occlusion and shadows, we show that eclipses can be described and reasoned about using mereotopological relations, and that these formalisms can be used to disprove some commonsense misconceptions about the nature of celestial phenomena.

Introduction

Qualitative Spatial Reasoning (QSR) (Ligozat 2011) is the subfield of Knowledge Representation in Artificial Intelligence that develops and applies formal representations of qualitative knowledge about spatial phenomena. QSR formalisms have found a range of applications in areas such as Geographical Information Systems, Architecture Design, Cognitive Vision and Robotics etc (Cohn and Renz 2008; Bhatt et al. 2011; Bhatt, Schultz, and Freksa 2013). However, to the best of our knowledge, qualitative theories about space have not been significantly applied in education, e.g., specifically as a tool for checking students' conceptions of the physical world, or in the modeling of astronomical events. In this paper we describe the initial steps towards generating a formalisation of a basic astronomical event (eclipses) that could be used in an autonomous tutorial system to both interact with the student's understanding of the concept and to address particular learning errors.

Several authors point out an apparent prevalence of alternative conceptions about the causes of natural events that contradict basic scientific knowledge, even after formal instruction on the subject (Libarkin and Kurdzie 2001). This has been linked to current educational theories which claim that people obtain new knowledge based on their existing beliefs (D.Bransford, L.Brown, and R.Cocking 2000) and, when existing beliefs clash with new knowledge, the former prevails over the latter. In naïve astronomy, there is a common misconception that the phases of the Moon are due to lunar eclipses (Bailey and Slater 2004). The formalisation presented below, although preliminary, provides a way to mitigate this common misconception.

The work presented here belongs to a family of QSR formalisms which make explicit the notion of viewpoints in their ontologies (as described in Section § **Related Work**). Here we apply one such method to model a few basic concepts (linking occlusion relations and visual appearance) of eclipses as described in Section §**Eclipses**. The formalism used in the paper is an occlusion calculus defined upon a mereotopology as introduced in Section §**Region Occlusion Calculus**. The main results of this paper are presented in Section §**A qualitative formalisation of eclipses**. The formalism thus defined is implemented within the Constraint Logic Programming system CLP(QS) in our penultimate section.

Related Work

Much work in Qualitative Spatial Reasoning does not explicitly model the observer's viewpoint. Without including a *point of view* or *observer location* as one of the variables in a formalisation, a theory is limited in its capacity to model perception. This limitation precludes the ability to reason about concepts and inferences involved in naïve astronomy.

There are, however, a few QSR formalisms that consider viewpoints when accessing whether a particular spatial relation holds or not. Most of these formalisms have spatial occlusion (or motion parallax) as a key aspect of their ontology. Spatial occlusion occurs when an object is located between another object and the observer's viewpoint; it is one of the primary cues used by the human perceptual system to construct a 3D interpretation of the visual world as it provides an estimate of relative distances (Randell, Witkowski, and Shanahan 2001). Perhaps the first qualitative formalisation of spatial occlusion was proposed in (Petrov and Kuzmin 1996) where a set of axioms is designed to constrain a point-based notion of occlusion. Assuming 2D convex objects, rather than points, (Galton 1994) proposes the *Lines-of-Sight* calculus that represents the relative positions between pairs of bodies as seen from a viewpoint. Based on this idea, the *Region Occlusion Calculus* (ROC) (Randell, Witkowski, and Shanahan 2001) defines occlusion and image parallax within a mereotopological theory. More recently, (Guha, Mukerjee, and Venkatesh 2011) proposed a set of 14 occlusion relations making explicit the distinctions of whether the observed objects are fragmented or not, and whether the occluder is a moving object or part of the background. (Tassoni et al. 2011) develops a 3D qualitative formalism about visibility that has occlusion at its kernel. (Bhatt, Lee, and Schultz 2011) propose a declarative spatial resoning system where mixed qualitative-quantitative spatial reasoning may be (declaratively) performed in a logic programming setting; the system can be used to formally reason about a range of spatial representations, including for instance, visibility and occlusion relations such as in (Tassoni et al. 2011).

Occlusion has also been recently defined within an interval algebra in (Ligozat, Santos, and Samghabadi 2015) providing the appropriate tools for operating consistency checking in qualitative constraint networks.

In previous work, we proposed a dynamic formalism about occlusion in which qualitative changes observed by a mobile robot are the building blocks of the system (Santos 2007). Later, (Souchanski and Santos 2008) refined this formalism within a *reasoning about actions and change* framework, in order to facilitate autonomous inferences about the behaviour of other agents (as observed from a viewpoint). *Region Occlusion Calculus* has recently been applied within a Bayesian filter in (Santos et al. 2016) to generate an efficient algorithm for qualitative self-localisation for a mobile robot.

Although there is some work on formalising the perception of shadows by means of a Qualitative Spatial Reasoning theory (Santos, Dee, and Fenelon 2009), there is currently no literature describing attempts to represent the observation of astronomical phenomena, such as eclipses, within a QSR framework. The representation and simulation of physical systems at a qualitative level has long been one of the goals of the Qualitative Reasoning (QR) field (Bredeweg and Struss 2004; Kleer 1990). However, to the best of our knowledge, celestial eclipses (or other astronomical occlusion-related events) have not yet been targeted by this community. These phenomena can be seen as a special case of shadow casting, and this paper is our attempt to provide the framework for describing and reasoning about them.

Eclipses

Since antiquity, eclipses have been well-understood astronomic phenomena. Periodical patterns of eclipses were known to the Babylonians and led in some cases to successful predictions of the occurrence and the type of a lunar eclipse (Neugebauer 1952). A full mathematical characterisation (and prediction of both lunar and solar eclipses and, in the latter case of the precise visibility area) was possible only after the work of Newton, however. One particular contingency makes solar eclipses geometrically interesting: the ratio Moon-diameter/Moon-distance from Earth is approximately the same as the ratio Sun-diameter/Sun-distance from Earth, which means that the Moon can almost perfectly occlude the Sun (in this case we say that it is an eclipse of *magnitude one*). Due to the slightly elliptical nature of orbits, these distances fluctuate. So at times the Moon may more than completely hide the Sun as it is located on its orbit at a distance smaller than the one required for the perfect occlusion ratio. The extent of the occlusion accounts for the length of the eclipse (this is the case of an eclipse of magnitude greater than one). In other cases, the Moon is more distant and thus cannot fully occlude the Sun, giving rise to annular eclipses (this is an eclipse of magnitude less than one). In an annular eclipse the Moon obscures the centre of the solar disk, but not the whole disk, giving a remarkable bright halo effect sometimes called the "*ring of fire*".

The various states of a solar eclipse are shown in Figure 1. A partial eclipse of the sun occurs when the observer is located at the penumbra region (depicted in blue in Figure 1), a total eclipse occurs when the observer is in the umbra region (coloured in brown in Figure 1) and an annular eclipse happens when the observer is in the antumbra region (depicted in the colour green). When the observed images of the Moon and the Sun are seen as in contact for the first time is an event known as *First Contact*; the case when the Moon and the Sun are in contact for the last time during an eclipse is called Fourth contact. Second and Third contacts occur when the borders of the Sun and Moon images (as seen from a viewpoint) meet for the second and third times (as shown in Figure 1). These events will be defined in terms of mereotopological concepts below.

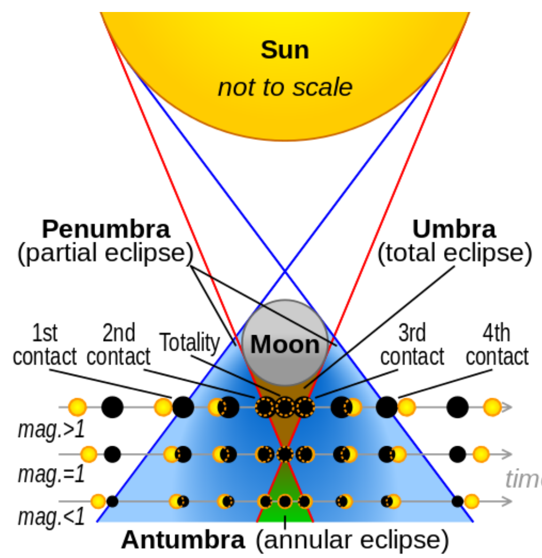


Figure 1: Visualisations of an eclipse. Figure adapted from `https://en.wikipedia.org/wiki/Eclipse`, accessed on Feb. 23, 2016.

Region Occlusion Calculus

The basic spatial theory used in this work is the Region Occlusion Calculus (ROC) (Randell, Witkowski, and Shanahan 2001), which is an extension of the Region Connection Calculus (RCC) (Randell, Cui, and Cohn 1992). RCC is a first-order axiomatisation of spatial relations based on a reflexive, symmetric and non-transitive dyadic primitive relation of *connectivity* ($C/2$) between two spatial regions. Informally, assuming two regions x and y, the relation $C(x, y)$, read as *"x is connected with y"*, is true if and only if the closures of x and y have at least one point in common.

Assuming the $C/2$ relation, and two spatial regions x and y, the following base relations can be defined: *disconnected from* (DC), *part of* (P), *equal to* (EQ), *overlaps* (O); *partially overlaps* (PO); *externally connected* (EC); *tangential proper part* (TPP); *non-tangential proper part* ($NTPP$). RCC also includes the inverse relations of P, TPP and $NTPP$, which are represented by a capital 'I' appended to the relative relation: PI, $TPPI$ and $NTPPI$.

The set constituted by the relations DC, EQ, PO, EC, TPP, $NTPP$, $TPPI$, and $NTPPI$ is the jointly exhaustive and pairwise disjoint set (JEPD) usually referred to as RCC8. The continuous transitions between the RCC8 relations, for two regions x and y, are shown as a *conceptual neighbourhood diagram* (CND) in Figure 2. By continuous transitions we mean that in between adjacent vertices of the graph there can be no other possible relation qualifying the state of the two regions. That is, assuming that the objects move continuously on the plane, these are the only transitions that are possible.

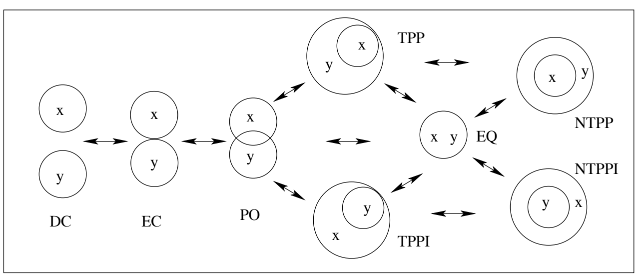


Figure 2: The RCC8 relations and their conceptual neighbourhood diagram (Randell, Cui, and Cohn 1992).

Let a and b be two physical (possibly non-convex) bodies, and ν an observer viewpoint. Using RCC8 relations, along with the primitive relation $TotallyOccludes(a, b, \nu)$ (which stands for "a totally occludes b with respect to the viewpoint ν"), the Region Occlusion Calculus (ROC) (Randell, Witkowski, and Shanahan 2001) defines the 20 base JEPD relations representing the various occlusion relations between two bodies. ROC distinguishes the occupancy regions of bodies and their images (or projections) from the viewpoint of an observer by assuming two functions: the function $region(a)$, which maps a body a to its 3D occupancy region, and the function $image(a, \nu)$ that maps a body a to the body's 2D projection, as seen from a viewpoint ν. The viewpoint in ROC is modelled as a pinhole camera whose parameters are not relevant here.

Figure 3 shows a graphical representation of the ROC relations between two bodies, represented as a white and a shaded region. In this figure, the shaded region corresponds to the first argument, and the white region to the second argument of ROC relations. For instance, the relation $PartiallyOccludesTPP(a, b)$ is depicted with the shaded object a occluding the white object b, while the 2D projection of the shaded object is a tangential proper part (TPP) of the 2D projection of the white object. It is worth noting that the relations on *mutual occlusion* only occur if and only if at least one of the objects is non-convex. ROC also defines a conceptual neighbourhood diagram (introduced in (Randell and Witkowski 2002)) that we do not present in this paper for brevity.

It is worth pointing out also that the "I" in the relations $TotallyOccludesTPPI(a, b, \nu)$ and $TotallyOccludesNTPPI(a, b, \nu)$ represents the inverse of TPP and $NTPP$, respectively; so, for instance, $TotallyOccludesTPPI(a, b, \nu)$, means that the body a totally occludes the body b, but $image(b)$ is the tangential proper part of $image(a)$ (i.e., $TPPI(image(a, \nu)$,-$image(b, \nu))$). The superscript "-1" in some ROC relations represents the inverse of the occlusion part of the relation.

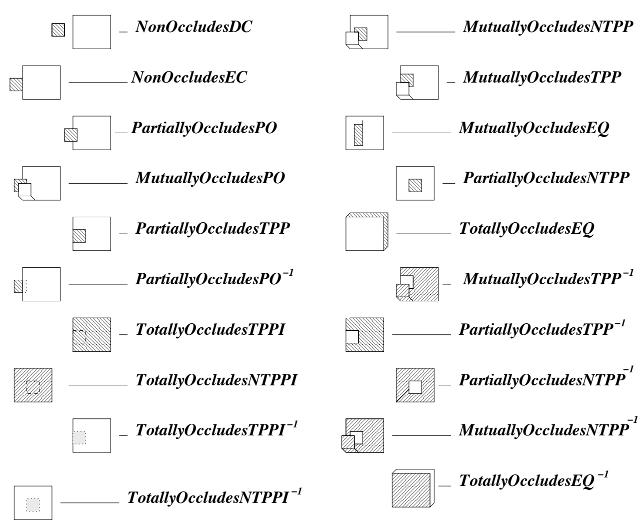


Figure 3: ROC relations between two objects (white and shaded regions).

As we are dealing with eclipses and therefore with celestial bodies, there are certain ROC relations which cannot hold in our situation. These relations are those which involve mutual occlusion.

Relative Positions

As well as the 20 ROC relations, this work assumes observer-relative positions of pairs of objects by means of the relations $Left$ and $Right$. Given two distinct bodies a and b and a viewpoint ν (where ν, in this case, is any viewpoint on Earth observing the eclipse) the relative positions between a and b with respect to their centroids are as follows:

- $Left(a, b, \nu)$, representing the fact that "a is on the left of b from the viewpoint ν";

Table 1: ROC relations representing eclipse states. The numbers in the left column are abbreviations used in the remainder of this paper.

1	$NonOccludesDC(M, S, v) \wedge Right(M, S, v)$.
2	$NonOccludesEC(M, S, v) \wedge Right(M, S, v)$.
3	$PartiallyOccludesPO(M, S, v) \wedge Right(M, S, v)$.
4	$PartiallyOccludesPO(M, S, v) \wedge Left(M, S, v)$.
5	$NonOccludesEC(M, S, v) \wedge Left(M, S, v)$.
6	$NonOccludesDC(M, S, v) \wedge Left(M, S, v)$.
7	$TotallyOccludesEC(M, S, v)$.
8	$PartiallyOccludesTPP(M, S, v) \wedge Right(M, S, v)$.
9	$PartiallyOccludesNTPP(M, S, v)$.
10	$PartiallyOccludesTPP(M, S, v) \wedge Left(M, S, v)$.
11	$TotallyOccludesTPPI(M, S, v) \wedge Right(M, S, v)$.
12	$TotallyOccludesNTPPI(M, S, v)$.
13	$TotallyOccludesTPPI(M, S, v) \wedge Left(M, S, v)$.

- $Right(a, b, \nu)$, representing the fact that "a is on the right of b from the viewpoint ν".

A qualitative formalisation of eclipses

Qualitative reasoning about eclipses proves to be a difficult task (Sorensen 1999). It may even be at the heart of some inconsistent commonsense reasoning, such as the common belief that the phases of the Moon are caused by the shadow cast by the Earth on the Moon (Barnett and Moran 2002; Bailey and Slater 2004). In this section we tackle qualitative reasoning about eclipses by means of the Region Occlusion Calculus described in the previous section.

We first assume two constants: M and S representing, respectively, the Moon and the Sun. With these constants, and the ROC relations described above, the qualitative states of a solar eclipse can be described by the conceptual neighbourhood diagram shown in Figure 4, where the dark object represents the Moon and the bright one, the Sun. The numbers assigned to each state in Figure 4 are abbreviations of conjunctions of ROC and Relative Position relations representing eclipse states, as shown in Table 1. In all of these diagrams and formulae the viewpoint v, if not explicitly stated, is assumed to be a location upon the surface of the Earth.

It is worth noting that the transition from 3 to 4 in Figure 4 represents a partial eclipse; the transitions from 3 to 7 and then from 7 to 4 represent an eclipse of magnitude equal to 1 (cf. Figure 1). Similarly, the transitions $3 \rightarrow 8 \rightarrow 9 \rightarrow 10 \rightarrow 4$ and $3 \rightarrow 11 \rightarrow 12 \rightarrow 13 \rightarrow 4$ represent, respectively, eclipses of magnitude greater than 1 ($mag. > 1$) and less than 1 ($mag. < 1$).

This formalisation includes all the states of an eclipse, as shown in Figure 1 and also represented in Figure 5 (the numbers in Figure 5 stand for the relations in Table 1).

Figure 5 also provides us an opportunity to define the astronomical terms in Figure 1 by means of ROC relations as follows:

- *Penumbra (or partial eclipse)* happens in the region where $PartiallyOccludesPO(M, S, v) \wedge Right(M, S, v)$ or $PartiallyOccludesPO(M, S, v) \wedge Left(M, S, v)$;
- *Umbra (or total eclipse)* happens in the region where $TotallyOccludes(M, S, v)$ holds (i.e. in 7, 11, 12 and 13);
- *Antumbra (or annular eclipse)* happens in the region where $PartiallyOccludesTPP(M, S, v) \wedge Right(M, S, v)$, or $PartiallyOccludesNTPP(M, S, v)$, or $PartiallyOccludesTPP(M, S, v) \wedge Left(M, S, v)$;
- *First Contact* can be defined as $NonOccludesEC(M, S, v) \wedge Right(M, S, v)$ (state 2 in Figure 5);
- *Second Contact* can be defined as $TotallyOccludesTPPI(M, S, v) \wedge Right(M, S, v)$ (state 11 in Figure 5);
- *Third Contact* can be defined as $TotallyOccludesTPPI(M, S, v) \wedge Left(M, S, v)$ (state 13 in Figure 5);
- *Fourth Contact* can be defined as $NonOccludesEC(M, S, v) \wedge Left(M, S, v)$ (state 5 in Figure 5).

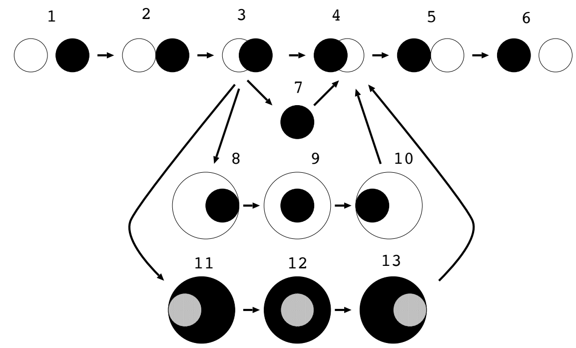


Figure 4: Solar eclipse as occlusion relations.

Thus far we have shown that the formalisation here encompasses the phenomena of solar eclipses. With a minimal amendment, we can also accommodate lunar eclipses. The ROC relations in Table 1 are described from the perspective of an observer on the surface of the Earth.

To model a lunar eclipse we need to consider shadows more explicitly. In a lunar eclipse, the Moon is in the shadow of the Earth, and no (pointlike) light source can see the shadows cast by objects that intercept light rays emanating from it (as observed by Da Vinci). Thus we have an analogous situation (the Earth occludes the Sun with respect to the Moon).

As an instance of a qualitative derivation within the proposed calculus, with the formalisation introduced above it is straightforward to derive the negation of the common belief that the phases of the Moon are the result of eclipses. Assume, reasoning by *Reductio ad absurdum*, that the phases of the Moon are indeed due to lunar eclipses, then the Moon (M) and the Earth (E) must be in one of the occlusion relations $\{3, 4, 7, 8, 9, 10, 11, 12, 13\}$ with respect to

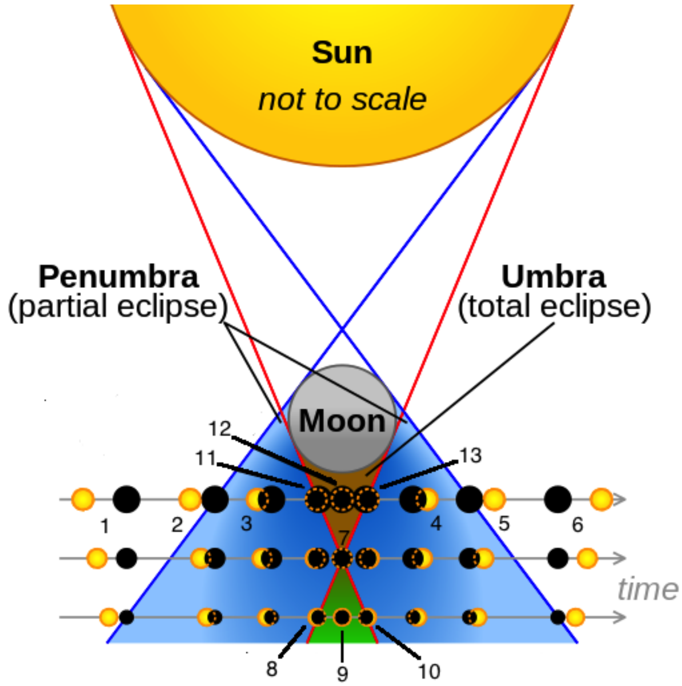


Figure 5: Solar Eclipse as occlusion relations: regions labelled with their ROC eclipse states from Table 1.

the illuminating surface of the Sun (viewpoint ν). Thus, from the definitions of the ROC relations we have that $C(image(M,\nu), image(E,\nu))$ (as presented in Section §*Region Occlusion Calculus*). However, in the case of a lunar phase it is a fact that $\neg C(image(M,\nu), image(E,\nu))$. The concluding step is a direct result of a qualitative model of lunar phases (not described in this paper).

A Declarative Implementation with CLP(QS)

In this section we present our implementation of our qualitative solar eclipse model in the Constraint Logic Programming system CLP(QS) (Bhatt, Lee, and Schultz 2011; Schultz and Bhatt 2012). Our implementation provides two key features:

- *Intelligent diagrams*: users can manipulate the objects in the diagram, and the system automatically updates other objects so that the qualitative spatial relations are maintained at all times;
- *Spatial Question / Answering*: users specify state constraints at both the domain level (solar eclipse states) and qualitative spatial level (topological and orientation relations between the Sun and the Moon) and CLP(QS) determines whether the constraints are consistent, and updates the intelligent diagram accordingly.

The interactive and dynamic aspects of intelligent diagrams (also referred to as *dynamic geometry*) makes them highly attractive for use in education domains such as teaching high-school level geometry (Winroth 1999). We implement our qualitative model in CLP(QS) and generate intelligent diagrams from two perspectives: (1) *top-down* and (2) *from-Earth*.

We define facts and rules for referring to solar objects from different perspectives, e.g. the following query specifies that the Moon and Sun in the top-down perspective are topologically *disconnected*:

```
object(type(moon), perspective(top_down), Moon),
object(type(sun),  perspective(top_down), Sun),
topology(rcc(dc), Moon, Sun).
```

For brevity, in the following we omit these type casting predicates when there is no ambiguity about perspective.

Top-down perspective. In the *top-down* perspective Earth is a *point*, the Moon is a *circle*, and the Sun is a *circle*. The Moon's orbit is a circle centred on Earth, such that the centre of the Moon is coincident to the orbital circle. Earth's orbit is a circle concentric with the Sun, such that Earth is coincident to the orbit.

```
point(Earth),
circle(MoonOrbit),
centre(MoonOrbit, Earth),

circle(Sun),
circle(SunOrbit),
incidence(concentric, SunOrbit, Sun),
size(larger, SunOrbit, Sun),

circle(Moon),
centre(Moon, MoonCentre),
incidence(coincident, MoonCentre, MoonOrbit),
incidence(coincident, Earth, SunOrbit),
size(smaller, Moon, Sun),
```

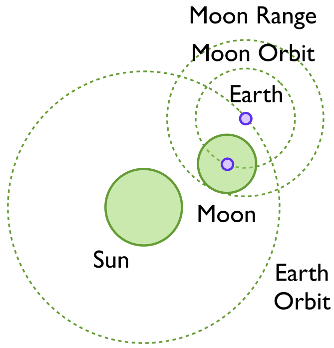
We add further constraints on the relative size and topology of our solar objects and their orbits. Firstly, the Earth is exterior to the Moon. Secondly, we want that the Moon and Sun never overlap. We thus define a Moon range circle, concentric with the Earth, such that the Moon is a tangential proper-part, and the Sun is disconnected (Fig. 6a).

```
incidence(exterior, Earth, Moon),
circle(MoonRange),
centre(MoonRange, Earth),
topology(rcc(tpp), Moon, MoonRange),
topology(rcc(dc), Sun, MoonRange),
```

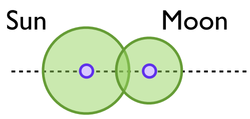
From-Earth perspective. In the *from-Earth* perspective the Moon and Sun are *circles* such that the line between their centroids is horizontal (Fig. 6b).

```
circle(Moon), circle(Sun),
centre(Moon,MoonCentre),
centre(Sun,SunCentre),
orientation(horizontal,line(MoonCentre,SunCentre)),
```

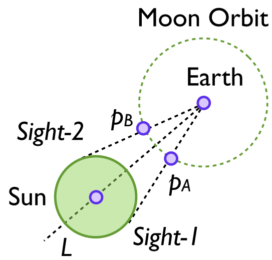
Connecting perspectives. To qualitatively relate the top-down and from-Earth perspective we need to relate the size and relative positions of the solar objects. That is, when we manipulate the objects in one diagram we want the objects



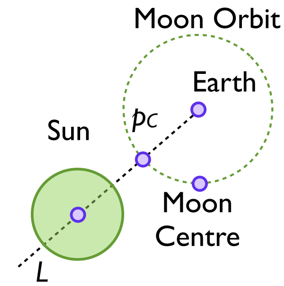
(a) Top-down perspective.



(b) From-Earth perspective.



(c) Size of Sun relative to Moon orbit (i.e. distance between p_A, p_B).



(d) Perceived distance between Moon and Sun from Earth.

Figure 6: Implementing solar eclipse model in CLP(QS).

in the other diagram to also correctly change (e.g. if we resize and move the Sun in the top-down diagram then the Sun in the from-Earth diagram should also automatically change so that the diagrams remain consistent with each another).

Firstly, we set the Moon size in both perspectives to be equal.

```
object(type(moon), perspective(top_down), MoonTopDown),
object(type(moon), perspective(from_earth), MoonFromEarth),
size(equisized, MoonTopDown, MoonFromEarth).
```

Next we constrain the relative Sun size, that is, we need to relate the radii of the circles representing the Suns in the top-down and from-Earth perspectives. We can not simply constrain the Suns' radii to be equal, otherwise the Sun in the from-Earth perspective would be far too large relative to the Moon. Instead, we need to express the *perceived* diameter of the Sun relative to the Moon in the top-down perspective, and use this diameter to equal the diameter of the Sun in the from-Earth perspective.

To accomplish this, in the top-down diagram we add two lines-of-sight from Earth to either side of the Sun; these represent the left-most and right-most points of the Sun that are visible from Earth, respectively. Consider the points p_A, p_B where these lines-of-sight intersect the Moon's orbit (Fig. 6c). The length of the arc between p_A, p_B along the Moon orbit circle is the perceived diameter of the Sun relative to the size of the Moon. Thus, the diameter of the Sun in the from-Earth perspective is set to equal the length of the arc along the Moon orbit circle between points p_A, p_B.

Let L be the line between the Earth and the Sun's centre point in the top-down perspective. We define two sight lines (Sight-1, Sight-2) such that (a) they are tangent to the Sun, (b) one of the end-points of each sight line is coincident to Earth, (c) the other end-point is both coincident to the Sun, and either to the left or right of L, respectively for each sight line. We then define intersection points p_A, p_B between the Moon's orbit and each sight line. Let r be the radius of the Moon orbit circle. The length of the arc of radius r, centred on the Earth, from p_A to p_B is equal to the diameter of the Sun in the from-Earth perspective (Fig. 6c).

```
object(type(sun), perspective(top_down), SunTopDown),
object(type(sun), perspective(from_earth), SunFromEarth),
...
L = line(Earth, SunCentreTopDown),
Sight1 = line(Earth, SightPoint1),
Sight2 = line(Earth, SightPoint2),

orientation(tangent, Sight1, SunTopDown),
orientation(tangent, Sight2, SunTopDown),
incidence(coincident, SightPoint1, SunTopDown),
incidence(coincident, SightPoint2, SunTopDown),
orientation(left,  SightPoint1, L),
orientation(right, SightPoint2, L),

point(PA), point(PB),
incidence(coincident, PA, MoonOrbit),
incidence(coincident, PB, MoonOrbit),
incidence(collinear, PA, Sight1),
incidence(collinear, PB, Sight2),

size(equal, arc(MoonOrbit, PA,PB),
           diameter(SunFromEarth)).
```

Finally, we constrain the relative distance and orientation of the Earth and the Moon, that is, if the position of the Moon in the top-down diagram is changed then the relative position between the Sun and Moon in the from-Earth perspective must also change (and vice versa).

In the top-down diagram let p_C be the intersection point between L and the Moon's orbit, and again let r be the radius of the Moon's orbit. The length of the arc with radius r, centred on the Earth, from p_C to the Moon's centre is the perceived distance between the centre of the Sun and Moon in the from-Earth perspective.

In the top-down diagram the Moon's centre point is either left of, collinear to, or right of an "arrow" pointing from Earth to the centre of the Sun (i.e. L). This relative orientation relation between L and the centre of the Moon is constrained to be the same as the relative orientation of the centre of the Moon and a vertical "arrow" from the centre of the Sun, pointing upwards, in the from-Earth diagram (Fig. 6d).

```
incidence(coincident,PC,MoonOrbit),
incidence(collinear,PC,L),
centre(Moon, MoonCentre),
size(equal, arc(MoonOrbit,PC,MoonCentre),
     line(SunCentreFromEarth, MoonCentreFromEarth)),
orientation(Relation, MoonCentreTopDown, L),
orientation(Relation, MoonCentreFromEarth, L3),
```

Defining eclipse states. We implement all states including *penumbra*, *first contact*, etc. and magnitude relations. For example, the penumbra occurs when the Sun is externally connected (ec) or partially overlapping (po) the Moon in the from-Earth perspective. First-contact occurs during penumbra when the centre of the Moon is to the right of the centre of the Sun. The magnitudes define the perceived relative size of the Moon and Sun in the from-Earth perspective.

```
state(penumbra) :-
  object(type(moon), perspective(from_earth), Moon),
  object(type(sun),  perspective(from_earth), Sun),
  (topology(rcc(ec), Moon, Sun);
   topology(rcc(po), Moon, Sun)).
```

```
state(first_contact) :-
  state(penumbra),
  object(type(earth),perspective(top_down), Earth),
  object(type(moon), perspective(top_down), Moon),
  object(type(sun),  perspective(top_down), Sun),
  centre(Moon, MoonCentre),
  centre(Sun,  SunCentre),
  orientation(right, MoonCentre, line(Earth, SunCentre)).
```

```
magnitude('>1') :-
  object(type(moon), perspective(from_earth), Moon),
  object(type(sun),  perspective(from_earth), Sun),
  size(larger,Moon,Sun).
```

Spatial Q/A. Users can express Prolog queries about eclipse states and qualitative spatial relations in both perspectives seamlessly. For example, what is the topological relation between the Moon and Sun (from Earth's perspective) during the *umbra*?

```
?- state(umbra),
|  object(type(moon), perspective(from_earth), Moon),
|  object(type(sun),  perspective(from_earth), Sun),
|  topology(Relation, Sun, Moon).
Relation = rcc(tpp);
Relation = rcc(ntpp);
Relation = rcc(eq);
false.
```

What magnitude corresponds with the antumbra?

```
?- state(antumbra),
|  magnitude(Magnitude).
Magnitude = '<1';
false.
```

If the Moon is to the *left* of sight line Sight-1, can the Moon and Sun partially overlap in the from-Earth perspective?

```
?- object(type(moon),perspective(top_down), MoonTopDown),
|  object(type(sight_1), perspective(top_down), Sight1),
|  orientation(left, MoonTopDown, Sight1),
|
|  object(type(moon), perspective(from_earth),
|       MoonFromEarth),
|  object(type(sun),  perspective(from_earth),
|       SunFromEarth),
|  topology(rcc(po), MoonFromEarth, SunFromEarth).
false.
```

Intelligent Diagrams. As users manipulate diagrams of either perspective, the objects in both diagrams are automatically updated so that all qualitative spatial relations are maintained. Thus, users can explore different configurations of objects and observe the relationship between the perspectives.[1] Moreover, at any stage the user can query the diagram via a Prolog query as above, or modify the diagram by enforcing spatial constraints, specifying states, etc.

Conclusion and Future Research

In this paper we have sketched a route to a qualitative theory of eclipses, building upon previous work in occlusion and shadow reasoning. The incorporation of viewpoints into reasoning about perception and shadows provides a route to understanding celestial phenomena in a new way. Our model allows the characterisation of our visual experiences of eclipses in terms of occlusion relations, and the prediction of visual experiences given occlusion relations.

We have gone on to implement this qualitative model for eclipses in terms of the Constraint Logic Programming system CLP(QS). This implementation provides *intelligent diagrams*, with which users can interact with the qualitative model by manipulating its objects, and *Spatial Question/Answering* about the domain modeled. These features facilitate a seamless interaction between users and the domain. This could be used as a tool for a hypothesis testing procedure in an educational context (in a similar way as described in (Forbus et al. 2005)). In this paper we have demonstrated the intelligent diagram aspects and the question answering aspects, however the actual use and evaluation of this method in the classroom is a task for future research.

An interesting issue for a future work is to use a qualitative calculus in order to infer, given what is observed from Earth during an eclipse, what can be seen of the Earth-Moon system from any point on the surface of the Sun at the same time. For our purposes, the Sun can be modeled as an infinity of pointwise light sources, each of which can be considered a viewpoint. Each of these viewpoints sees the Moon fully occluding its own shadow cast on Earth, surrounded by a penumbral area which is the union of the shadows cast by all other points on the solar surface. No point in the Sun, thus, sees the umbra, and all points see a portion of the penumbra, which is partly occluded by the Moon itself. These viewpoints belong to different visibility classes, defined by the ROC relations holding on Earth.

[1] We use FreeCAD as a front end for the intelligent diagrams.

In this paper we assumed ROC relations as defined for single observation points. However, the Earth is sufficiently large to support salient parallax effects during the occurrence of an eclipse. Totality is visible in a certain area (the shadow's path of totality), the surrounding areas only experience partial eclipses, and the areas further away see tangential eclipses or no eclipses at all. Similarly, along the path of totality, totality sets in at different times, so that occlusion relations differ at different places. An opportunity thus arises to consider the combination of the information obtained by the multiple viewpoints into a single (qualitative) description of the phenomena. This is also an issue left for future research.

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