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A Six-Stimulus Theory for Stochastic Texture

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Abstract. We report a six-stimulus basis for stochastic texture perception. Fragmentation of the scene by a chaotic process causes the spatial scene statistics to conform to a Weibull-distribution. The parameters of the Weibull distribution characterize the spatial structure of uniform stochastic textures of many different origins completely. In this paper, we report the perceptual significance of the Weibull parameters. We demonstrate the parameters to be sensitive to orthogonal variations in the imaging conditions, specifically to the illumination conditions, camera magnification and resolving power, and the texture orientation. Apparently, the Weibull parameters form a six-stimulus basis for stochastic texture description. The results indicate that texture perception can be approached like the experimental science of colorimetry.

Keywords: texture, natural image statistics, fragmentation

1. Introduction

An object in front of us will fragment the field of view into regions inside and outside the object. The region outside the foremost object or detail may be fragmented again by the presence of another object or another detail behind it, and so on until in any direction the field of view is bounded. When the scene is sufficiently fragmented by objects or details, the scene becomes stable and addition of a new object will not significantly change its appearance. Then, the mixture of cast shadows, edges and reflections result in an integrated view we call texture. Our definition emphasizes a different aspect of texture than the definition by Julesz (1981) which had the perception of the regular patterns of textons as its prime target. We restrict to the spatial stochastic aspect of texture, that is the statistical organization of the texture elements. The definition captures both the spatial statistics of the composition of a scene, and the stochastic structure of a material surface observed as object texture.

Research has not yet lead to a suitable basis for texture perception. Julesz (1981) in this respect focused on a single aspect of texture, the texture element or texton. He was able to pinpoint one of the aspects of texture. Nowadays, more research concentrates onto distinct aspects of texture. Zhu et al. (2000) gives conditions for a sufficiently rich statistical description of texture. Dana et al. (1999) concentrates on the bi-directional reflectance aspects of three-dimensional textures (Dana and Nayar, 1999; Cula and Dana, 2001). Liu and Tsin (2004) concentrate on the spatial layout of regular textures, as modelled by invariant Frieze patterns. Suen and Healey (2000) and Pont and Koenderink (2004) examine the photometric and chromatic dependence of observed texture on illumination and viewing angle.

We concentrate on the spatial statistics of stochastic textures. We maintain that stochastic textures can be observed on a basis similar to the tri-stimulus basis for color. In the RGB-color representation by the human sensory system, each of the three elements produces a value as the outcome of an integration process over part of the spatial and wavelength spectrum. Similarly, we define stochastic texture on an orthogonal basis, which is the result of integrating over a spectrum of spatial fragments.

Texture is a stochastic descriptor of the local spatial size distribution as observed. Using an infinitely precise sensor we would see the details around us at extremely many scales, yielding a confusingly amount of information useless to the observer. To escape the influx of so much information, the spatial resolution of sensors must be limited to a scale of detail. The largest reduction in information is at the retina where the outside world is integrated over discrete sensory cells of finite resolution. The finite cell size will impose spatial coherence to the picture. In this reduction, as we have analyzed in Geusebroek and Smeulders (2003), the statistical distribution of the spatial size distribution will generally deviate from the power-law of fractals (Mandelbrot, 1983). The observation on the retina by a receptive field with finite extent imposes a distribution of the Weibull type in the observed texture.

In a Weibull type distribution, $f(y) = C \exp(-|(y-\mu)/\beta|^{\gamma})$, the parameters μ, β, and γ represent the center, width, and shape of the distribution, and C being a normalization constant. The shape parameter, γ, ranges from 0 to 2 (Gnedenko and Kolmogorov, 1968). For $\gamma = 2$ the Weibull distribution is equivalent to the normal distribution, and for $\gamma = 1$ a double exponential. For small values of γ, the distribution is close to the symmetric power-law, given by $f(y) = \frac{1}{2}\delta|y|^{-\delta-1}$, the exponent δ indicating the fractal dimension (Mandelbrot, 1983). The fragmentation of the image, generated by the edges between the objects in the scene, set off the spatial statistics of the view on the scene.

The spatial statistics as such are the result of an image formation process equivalent to the process generating fragmentation in grained objects (Brown, 1989; Brown and Wohletz, 1995). The sequential fragmentation process is essentially the same process that distributes surface details in homogeneous textures (Geusebroek and Smeulders, 2002). As the visual field of view is fragmented by the opacity of objects and integrated over the sensory receptive fields, we anticipate to observe a Weibull distribution for any natural texture. Quantitative evidence for the fragmentation process as generator for natural image statistics is given in Geusebroek and Smeulders (2003).

In this paper, we derive a 6-stimulus basis for texture perception from the statistical process of sequential fragmentation. We start our analysis by introducing the Weibull sequential fragmentation process (Section 2) as derived in Geusebroek and Smeulders (2002). Main contribution in this paper is the empirical derivation of the perceptual significance of the Weibull process (Section 3). We elaborate on the experimental investigation given in Geusebroek and Smeulders (2003) on the marginal statistics of texture, and investigate the Weibull parameters as function of orientation in Section 4. As a consequence of the analysis in Sections 3 and 4, we conclude that the spatial stochastics of a two-dimensional texture is completely characterized by the Weibull parameters. A second consequence is a minimum dimensionality of nine parameters to describe stochastic texture, identical to the experimental result of Suen and Healey (2000), which is indicated by the orthogonal Weibull parameters.

2. The Weibull Sequential Fragmentation Process

As a direct implication of causality, we consider that small details are occurring more often in an image than large structures (Koenderink, 1984). Diffusion of numerous small structures will result in fewer large structures. Inversely, increasing magnification at large structures will resolve many smaller structures. One may rephrase the statement in that, when resolving power increases, large structures will break-up into new structures, of which some of them are relatively large, but most of them will be small details. Hence, the histogram of contrasts for one structure typically shows a power-law distribution,

$$f(x' \to x) = \left(\frac{x}{\beta}\right)^{\gamma-1}. \tag{1}$$

where $x' \to x$ indicates the resolving of the structure x' into smaller structures of size x. The parameter β is related to the average structure contrast.

When more structures are added to the scene, the image will be fragmented into various patches, each giving rise to an edge of varying contrast. The histogram of the contrasts of the patches is the results of integrating over the various power-laws caused by every edge, as derived in Geusebroek and Smeulders (2002),

$$n(x) = c \int_x^{\infty} n(x') f(x' \to x)\, dx' \tag{2}$$

where $n(x)$ indicates the number of pixels with response magnitude between x and $x + dx$, contributed by all edges with contrast $x' > x$. The integration over a sufficient number of power-laws yields a Weibull distribution,

$$n(x) = \frac{1}{\beta}\left(\frac{x}{\beta}\right)^{\gamma-1} e^{-\frac{1}{\gamma}(\frac{x}{\beta})^{\gamma}}. \quad (3)$$

The Weibull shape parameter γ is related to the fractal dimension of the image, $D_f = \gamma + 1$ (Mandelbrot, 1983; Pentland, 1984). Note that D_f is a strictly spatial property of an image.

Mallat (1989), confirmed by Simoncelli (1999), empirically found the generalized Laplacian,

$$P(c) = ze^{-|\frac{c}{s}|^{p}} \quad (4)$$

to fit to the marginal statistics of wavelet coefficients. Here, c indicates the wavelet coefficient, and s indicates the variance. The exponent p is related to γ in Eq. (5). The generalized Laplacian is the integral form of the Weibull distribution. In integral form, the Weibull distribution given by

$$N(> x) = \int_x^{\infty} n(x)\, dx = e^{-\frac{1}{\gamma}|\frac{x}{\beta}|^{\gamma}} \quad (5)$$

indicates the relative amount of edges of (positive or negative) contrast larger than x.

In conclusion, causality implies a Weibull distribution of structure size. Hence, we have given a physical explanation for the empirical results as obtained by Mallat (1989) and Simoncelli (1999). A detailed experimental investigation of the fragmentation process as observed from natural images is given in Geusebroek and Smeulders (2003).

3. Perceptual Significance of the Weibull Process

The parameters of the Weibull distribution completely characterize the spatial layout of stochastically ergodic textures. Furthermore, the Weibull parameters generate a complete orthogonal basis for stochastic textures. The parameters of the Weibull distribution indicate the contrast in the image (β), the grain size (γ) relative to resolving power, and the global shape of the object (μ) as can be derived from shape from shading (Pentland, 1990). The perceptual meaning of these parameters is yet to be discussed. We adapt the perceptual properties as put forward by Tamura et al. (1978), namely *regularity*, *coarseness* or observation scale, *contrast*, *roughness*, and *directionality*.

Regularity. When the texture is Weibull distributed, the texture is stochastic of nature. If the Weibull hypothesis is rejected, the spatial distribution can be either power-law, or the distribution is generated by regular texture (Geusebroek and Smeulders, 2003). In the case of a power-law statistic, the texture consists of a clear foreground-background separation. In these cases, "texture" may be considered the wrong term for the image under consideration, we rather use "object" and "background". Alternatively, when the spatial statistics is caused by interference between the receptive fields and the spatial detail distribution, the observed distribution is often multi-modal. The effect is reflected in the maximum likelihood estimation for the Weibull γ parameter ($\gamma \gg 2$), caused by over fitting the tails of the distribution. In this case, the texture exhibits regularity in the spatial distribution, and more adequate analysis methods can be applied (Liu and Tsin, 2004).

Coarseness, or the scale of observation of the texture. We distinguish between *magnification* and *resolving power*. Observing a texture patch at increased magnification results in a coarser picture. In this case, scale-invariance may be achieved by adapting the receptive field size to the change in magnification. Hence, magnification is directly related to the scale of observation σ. Often an increase in magnification is accompanied by an increase in resolving power, resulting in a picture containing more (although magnified) of the small details of the texture than the original scene. Resolving power affects the distribution of detail size in the observed image. By increasing magnification while maintaining resolving power, no new details will be added, and the shape parameter γ will remain constant, see Fig. 1. By increasing resolving power, new details will be added. For pure fractal textures, a change in resolving power will not affect the spatial statistics. In general, natural images can be thought of as a "piled" set of fractals in fractals. Figure 2 depicts an example of rice taken at various distances. For large distance, a few grains of rice contribute to the local filter response, resulting in a value of $\gamma \approx 1.5$. Increasing resolving power together with magnification by decreasing the distance between preparation and lens will ultimately result in the imaging of a single grain of rice. Hence, the Weibull parameter will slowly converge towards power-law. For the results shown in Fig. 2, power-law

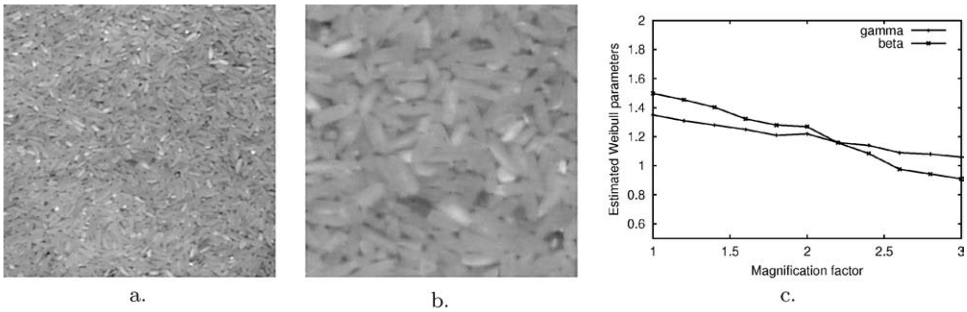

Figure 1. Weibull parameters as function of magnification without increasing resolving power (digital zooming). a. shows a rice image, b. the $3\times$ zoomed version. The Weibull parameter estimates are shown in c. From $2.6\times$ zoom, the γ-parameter remains constant. The β-parameter is decreasing due to reduced contrast in the zoomed field of view.

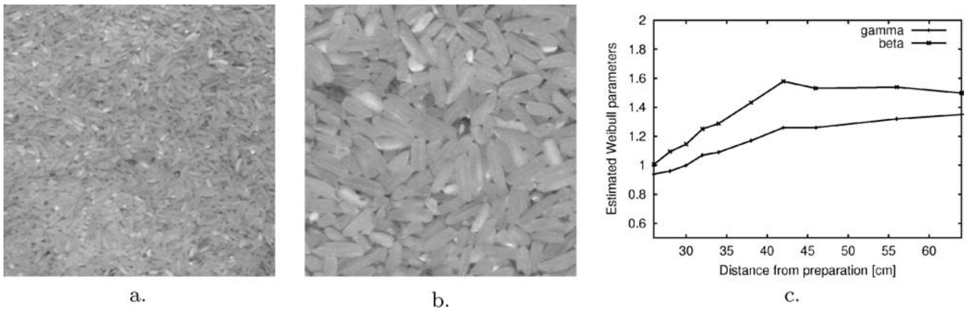

Figure 2. Weibull parameters as function of real magnification by decreasing distance to the preparation. a. shows the original image, b. the approximately $3\times$ zoomed version. The variation in the Weibull parameters is shown in **c**. Note the difference with Fig. 1.

is not yet reached as magnification is not sufficient to image a single grain of rice.

Contrast indicates the dynamic range of the gray-levels of the texture. Variations are caused by illumination intensity, and by surface height variations. Lighting variations or global shape variations are reflected in both the offset parameter μ and width parameter β of the Weibull distribution. The angle of incidence of the illumination, combined with approximately Lambertian reflection by the texture, will cause contrast variations indicating the shape of the object, similar to shape from shading (Pentland, 1990). The offset parameters μ capture these variation in first order, indicating a non-uniform illumination plane. Contrast differences to detailed surface height variations are mainly reflected in the width β of the Weibull distribution. Hence, after correction for a non-uniform illumination plane, the texture height variations are represented by β, see Fig. 3.

Roughness is determined by the grain size the texture constitutes of. With roughness we indicate the tactile properties of a surface, in our case as can be derived from the visual characteristics. Before assessing roughness, one has to correct for projective distortion (indicated by μ). Roughness is indicated by both the coarseness of the texture, as measured by γ, and by the contrast of the texture, as obtained from β. Where the contrast parameter β is indicative for the height variations of the texture, the γ parameter indicates the graininess of the texture. The combined evaluation of these parameters indicates the roughness properties of the texture. The entity may be related to the (three-dimensional) textons of which the texture is constructed (Julesz, 1981; Leung and Malik, 2001).

Directionality indicates the dominant orientation of the texture, effectuated by both the stochastic process placing the texture elements, and by the individual shapes of the textons. The Weibull basis does not include texton shape, hence we concentrate on the directionality caused by placement of the textons, disregarding texton elongation. Anisotropy in the grain size of the texture is reflected in the dominant direction θ_γ in the graininess parameter γ. Anisotropy in the texture shadows or albedo, hence the contrast, is reflected

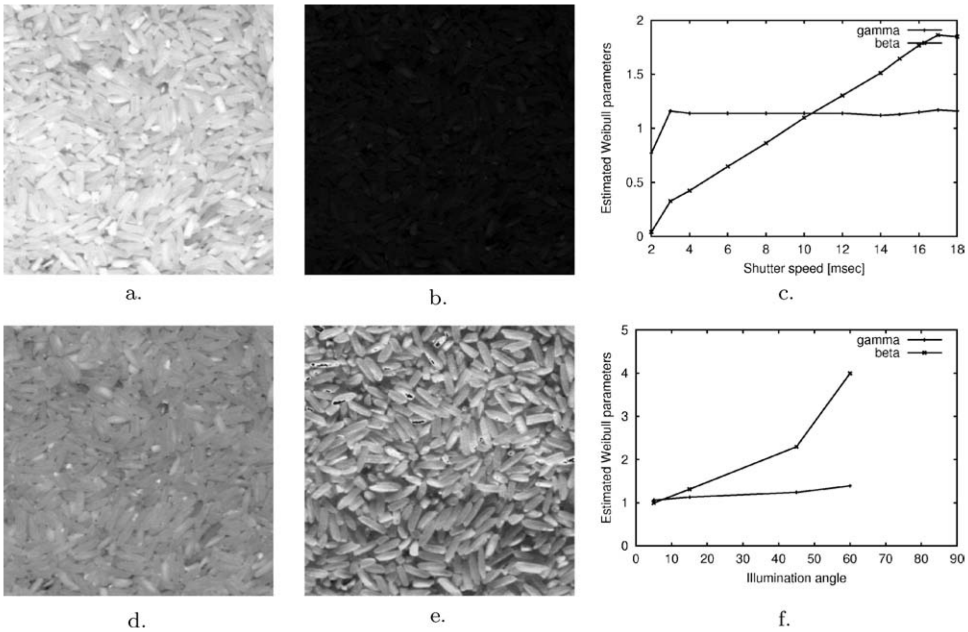

Figure 3. Weibull parameters as function of illumination intensity a–c and illumination angle d–f. a. shows the rice image at 18 msec shutter speed, saturating the white portions of the image. b. shows the rice image at 3 msec shutter speed, saturating the dark portions of the image. The Weibull parameters as function of shutter speed are illustrated in c. Within the dynamic range of the camera, the β-parameter is clearly linear with intensity. d. shows the rice image with frontal illumination, whereas e shows oblique illumination. Note the large variation in Weibull parameters, shown in f.

in the dominant orientation θ_β on the width β of the Weibull distribution. Hence, textures may exhibit two kinds of anisotropy, one caused by grain size, the other caused by contrast variations, both orientations being independent of each other (see Fig. 4). Consequently, at least two parameters are necessary to describe orientation, one for the principal direction of γ, and one for the principal direction of β.

In conclusion, the proposed Weibull basis conforms to previous results on the visual perception of texture features. We concentrated on parameters related to spatial texture layout rather than texton shape. We showed that at least two parameters are necessary to describe texture orientation, opposite to Tamura et al. (1978) who proposed one dominant orientation for texture perception.

4. Empirical Evaluation of a Stochastic Texture Basis

In Geusebroek and Smeulders (2003), we established for a collection of 50,000 images that the hypothesized distribution of the intensity differences is indeed of the Weibull type. Here, we are interested whether the two-dimensional Weibull parameters are sufficient in characterizing homogeneous, stochastically ergodic textures. The Curet collection is a proper testing ground (Dana et al., 1999). The collection consists of 61 materials, each taken under various illumination and viewing directions. The range of materials covers plaster, styrofoam, straw, corduroy, paper, brick, fur, and so on, effectively covering a range of materials with Lambertian reflection, with polarized reflection with highlights, to the mirror reflection of Aluminum foil.

Identical to the experimental setup in Geusebroek and Smeulders (2003), the intensity differences in a picture are assessed by scale normalized Gaussian derivative filters measured in 72 directions. The effective resolution of the system is given by the sigma of the filter coefficients, here set to correspond to a spatial observation resolution of 3 pixels at all experiments. The values of the Weibull parameters are estimated using the maximum likelihood method, and goodness-of-fit is evaluated by the Anderson-Darling statistic at a significance level $\alpha = 0.05$ (Filliben et al., 2002).

The results described in more detail in Geusebroek and Smeulders (2003) indicated the Weibull distribution to be present in 54 of the 61 Curet materials. The

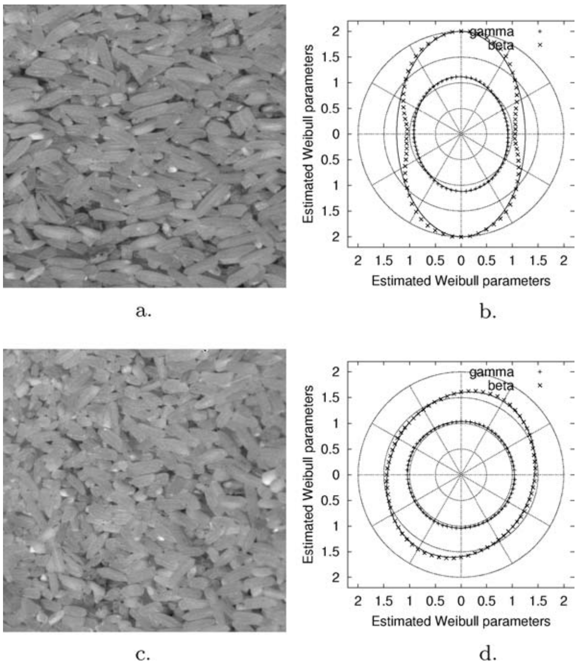

Figure 4. Weibull parameters as function of orientation. a. shows an image of combed rice, of which the Weibull parameters as function of filter orientation are shown in b. Both γ and β show an anisotropy due to the overall rice orientation. c. shows the same image from a 45 degree viewing angle, in which case the rice grain size γ appears isotropic, but shadow contrast β is not, as illustrated in d. The solid lines represent the best fitting ellipse through the data.

Weibull parameter estimates vary over the illumination and viewing direction according to the apparent size and directionality of the texture. When the texture is not stochastic but regular, such that there is repetition between the responses in an image, a Weibull distribution will no longer be found. However, as we are interested in the description of stochastic texture, these materials can be ignored.

Now we are in a position to access the completeness of the Weibull parameters in characterizing the spatial layout of stochastic textures. Previous results indicated the marginal statistics of texture images. Next we proceed with a full probability density. To evaluate a parametric description of the spatial stochastics of texture, we need to determine: (a) the completeness of the Weibull parameters based on the gradient filter, and (b) the consistency of the Weibull distribution over orientation of the gradient filter.

An important issue is whether the hypothesized Weibull distribution is general for receptive field measurements based on contrast. Hence, we experimentally examined the histogram of the responses to higher order Gaussian derivative filters. For the images conforming to a Weibull distribution of first-order derivative response, the statistics of a second-order derivative filter again is Weibull distributed. For the Curet database, we investigated the correlation between the Weibull parameters as function of derivative order. The Weibull shape parameter γ showed a high correlation between the first-order derivative and the estimate for the second-order derivative ($\rho = 0.96$). Likewise, the first- and third-order derivative yield a correlation coefficient of $\rho = 0.93$. Given the high correlation between the responses to various derivative filters, the texture spatial statistics per orientation are sufficiently exhausted by a single derivative filter. These findings support the coding of natural images by edge filters (Bell and Sejnowski, 1997), and empirically confirm the theoretical result obtained by Zhu et al. (2000) stating that the marginal distribution of filter responses characterize texture. We conclude that a single filter type, although measured in different orientations, is sufficient to assess the spatial statistics of stochastic textures.

The free parameters of the Weibull distribution have been evaluated as a function of orientation. The orbit of the shape parameter γ as function of the orientation of the derivative filter is found to be close to elliptic in all cases of uniform stochastic textures, with the aspect ratio indicating anisotropy in material roughness. This illustrates that the maximum and minimum $\gamma_{\max,\min}$ parameters are orthogonal descriptors of texture roughness indicative for detail size and the isotropy therein, see Fig. 5. The dominant orientation of the texture is indicated by θ_γ, the orientation of the elliptic orbit of γ. The intensity contrast of the texture is captured by the Weibull distribution width parameters $\beta_{\max,\min}$, representing height variations in the grains generating the texture. Again, it is observed that the β parameters of uniform texture describe closely an ellipse, indicating independence while they describe texture. The direction of anisotropy in contrast is indicated by θ_β.

The parameters γ and β are computed from local intensity derivatives orthogonal to the intensity-based μ parameter. In fact, the $\mu_{\max,\min}$ and θ_μ parameters of the Weibull distribution, together with the local intensity values completely characterize the intensity along the object's surface. In addition to this information about the local shape of the object (Pentland, 1990), the six

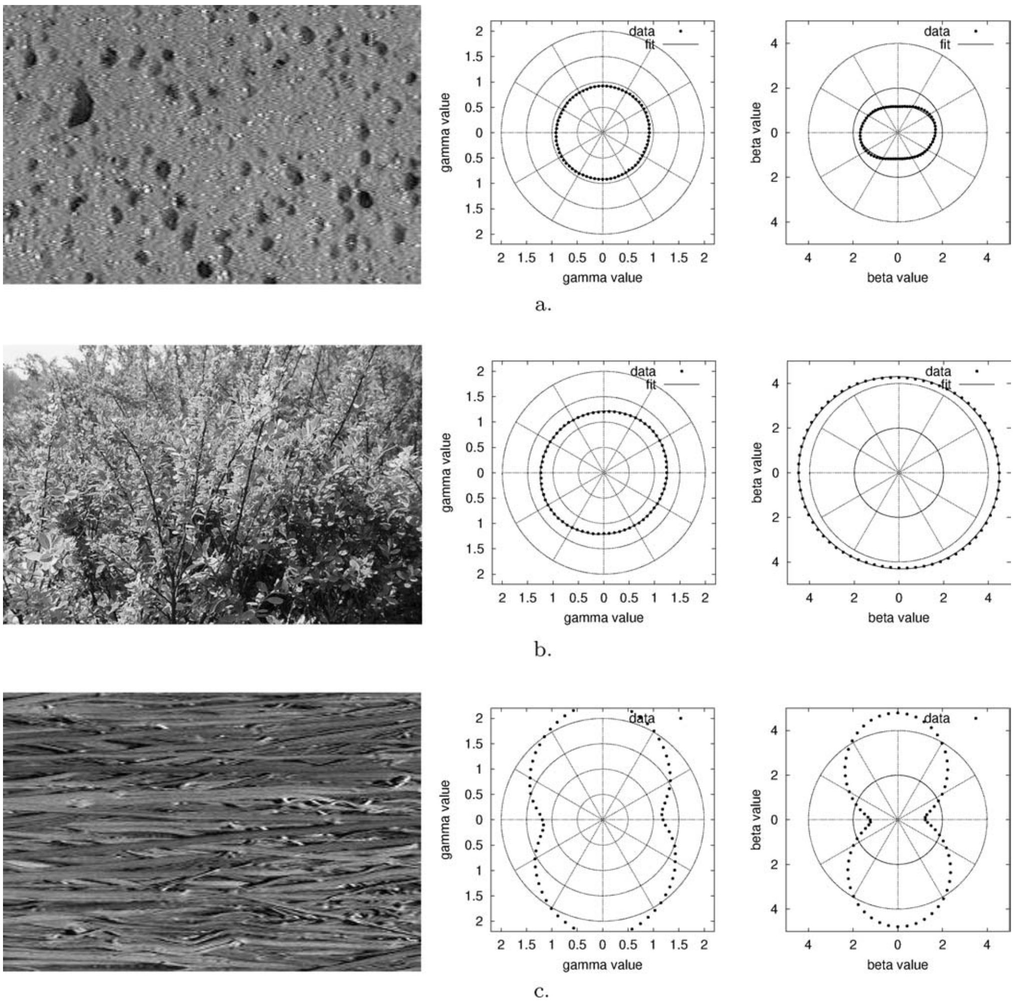

Figure 5. Plot of the six-parameter bases for a. sponge, b. shrub, and c. straw. The polar plot of γ for the sponge (a) shows an isotropic grained texture with $\gamma = 0.9$. The plot of β, indicating the texture contrast, is dominated by the cast shadows at the pores. For this case the longest axis indicates the direction of illumination. The shrub plot (b) for γ exhibits some anisotropy, yielding a longest axis of $\gamma_{\max} = 1.27$ in the 30°direction and $\gamma_{\min} = 1.20$ in the 120°direction (coarse grain size in that direction). The anisotropy is caused by the depth variation in the 120°direction. The plot for β exhibits anisotropy in the $-12°$ direction, which is approximately perpendicular to the direction of growth of the prominent twigs in the shrub. The more regular texture of straw c is rejected as being Weibull distributed in the y-direction, hence does not satisfy the six-parameter basis. The polar plots show the x-direction to conform to the Weibull process. When changing orientation toward the y-direction, the regular structure of the straw causes interference, and the Weibull parameters are over-estimated causing cusp's in the polar plots.

γ- and β-parameters form a basis to describe the spatial structure of homogeneous, stochastically ergodic texture.

Not all dimensions of the γ- and β-basis contain equally much information. The maximum value $\gamma_{\max}$ is proportional to the smallest grain size dimension, indicating texture roughness. The maximum value $\beta_{\max}$ is proportional to the largest grain height, measured as texture contrast. The aspect ratio $\gamma_{\max}/\gamma_{\min}$ indicates anisotropy in the grain size distribution, where θ_γ indicates the corresponding orientation. Furthermore, the aspect ratio $\beta_{\max}/\beta_{\min}$ indicates the anisotropy in the texture contrast, in the direction of θ_β. The two anisotropies as well as their orientations are observed as independent in many cases. The basis describes the remaining degrees of freedom for texture when the texton is the invariant. Compared to the experimental work by Suen and Healey (2000), for the case of the Curet

Table 1. Weibull basis for the Curet dataset.

Material	$\gamma_{\max}$	γ_{aspect}	θ_γ	$\beta_{\max}$	β_{aspect}	θ_β
Felt	2.04	0.87	5	1.45	0.67	90
Polyester	1.95	0.93	15	0.97	0.86	90
Terrycloth	1.72	0.97	25	3.64	0.74	80
Rough plastic	2.10	0.88	115	4.81	0.50	95
Leather	1.99	0.82	60	0.38	0.83	300
Sandpaper	2.07	0.87	235	1.07	0.77	260
Velvet	2.10	0.89	115	0.74	0.94	110
Pebbles	2.23	0.77	95	4.87	0.57	275
Frosted glass	2.11	0.91	85	0.18	0.86	80
Plaster_a	2.09	0.85	85	4.84	0.54	270
Plaster_b	1.95	0.90	40	6.21	0.57	270
Rough paper	2.15	0.90	85	0.73	0.48	85
Artificial grass	1.83	0.88	95	4.78	0.73	95
Roof shingle	1.54	0.92	155	2.44	0.87	90
Aluminum foil	1.34	0.87	135	6.12	0.76	300
Cork	2.00	0.86	30	2.39	0.77	85
Rough tile	1.73	0.83	0	0.60	0.72	100
Rug_a			Rejected			
Rug_b	1.63	0.96	45	2.52	0.77	85
Styrofoam	1.30	0.97	110	0.86	0.74	100
Sponge	0.94	0.96	145	1.70	0.69	90
Lambswool	2.09	0.80	210	1.06	0.64	75
Lettuce leaf	1.24	0.69	35	1.57	0.74	50
Rabbit fur	1.98	0.84	100	2.43	0.64	95
Quarry tile	1.96	0.82	80	0.29	0.56	85
Loofa	1.91	0.87	0	2.71	0.67	90
Insulation	1.27	0.92	135	2.29	0.59	95
Crumpled paper	1.65	0.78	50	4.76	0.50	75
Polyester zoomed	1.93	0.91	50	1.32	0.78	85
Plaster b zoomed	1.88	0.85	10	6.10	0.60	90
Rough paper zoomed	2.24	0.87	80	1.12	0.43	80
Roof shingle zoomed	1.34	0.93	80	3.17	0.76	85
Slate_a	1.61	0.88	45	0.66	0.88	135
Slate_b	1.45	0.97	90	0.37	0.81	40
Painted spheres			Rejected			
Limestone	0.96	0.91	120	0.68	0.75	85
Brick_a	1.85	0.94	75	2.52	0.75	95
Ribbed paper			Rejected			
Human skin	1.48	0.95	0	1.38	0.85	75
Straw			Rejected			
Brick_b	1.39	0.96	5	0.69	0.79	100

(*Continued on next page.*)

Table 1. (*Continued*).

Material	$\gamma_{\max}$	γ_{aspect}	θ_γ	$\beta_{\max}$	β_{aspect}	θ_β
Corduroy			Rejected			
Salt crystals	1.58	0.89	90	3.28	0.68	95
Linen	1.78	0.89	85	0.89	0.53	85
Concrete_a	1.80	0.86	75	6.09	0.59	265
Cotton	2.19	0.84	0	1.07	0.79	95
Stones	2.01	0.84	55	7.51	0.73	70
Brown bread	1.75	0.92	120	2.98	0.69	85
Concrete_b	1.78	0.89	115	3.27	0.62	95
Concrete_c	1.71	0.88	55	4.30	0.60	265
Corn Husk			Rejected			
White bread	1.25	0.91	50	1.42	0.65	85
Soleirolia plant	1.36	0.92	110	4.51	0.70	95
Wood_a	1.47	0.87	5	1.35	0.94	0
Orange peel			Rejected			
Wood_b	2.00	0.71	75	2.06	0.26	80
Peacock feather	0.89	0.82	30	1.49	0.66	230
Tree bark	1.52	0.84	0	3.33	0.85	160
Cracker_a	1.84	0.86	0	2.96	0.87	110
Cracker_b	1.14	0.96	80	1.24	0.76	90
Moss	1.04	0.95	20	1.97	0.78	75

See the pictures on http://www.cs.columbia.edu/CAVE/curet/

database, the provided basis can be considered as describing the 3D variation after 2D texture effects have been corrected. Hence, we theoretically confirm their result of 9 remaining degrees of freedom for 3D texture. Table 1 gives values for the six parameters as estimated on the Curet database.

5. Conclusion

In a world full of fractal objects and with observation by discrete cells on the retina, the dominant distribution of texture statistics follows the Weibull distribution. We have experimentally verified that the Weibull distribution characterizes spatial statistics for ergodic stochastic textures. Next to the intensity, color, and the three μ parameters for the shape of the object, the parameters γ and β of the Weibull distribution span a complete and orthogonal six-stimulus basis for uniform stochastic texture perception. The basis provided indicates that research on texture perception can be approached in a similar way as the experimental science of colorimetry. The metamers in our basis are formed by the absence of the texton (Julesz, 1981), as the basis only describes the spatial distribution of elements, hence is independent of the element shape.

We provide a minimum dimensionality to describe spatially ergodic stochastic textures. For a uniform textured region, a complete spatial description is obtained by the nine parameters discussed above. If the texture is isotropic, and after correction of a non-uniform illumination, five parameters are sufficient to describe the spatial characteristics. This theoretically derived minimum dimensionality corresponds well to the empirical work by Suen and Healey (2000), who found nine dimensions to capture the 3D variations in material textures.

The proposed six-stimulus basis is the result from the fragmentation of the image by edges between objects. Although we confirmed our theory for Gaussian intensity receptive fields, the fragmentation is anticipated for color receptive fields as proposed in Geusebroek et al. (2001). Future work includes the assessment of the six-stimulus parameters for color invariant descriptions. We intent to use the Weibull distribution to parameterize local image structure in large pictorial databases. Similarity between textures may now be expressed by similarity between the Weibull parameters, accelerating retrieval performance.

Acknowledgments

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