

# A Physical Basis for Color Constancy

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## Abstract

A fundamental problem in psychophysical experiments is that significant conclusions are hard to draw due to the complex experimental environment necessary to examine color constancy. An alternative approach to reveal the mechanisms involved in color constancy is by modeling the physical process of spectral image formation. In this paper, we aim at a physical basis for color constancy rather than a psychophysical one.

By considering spatial and spectral derivatives of the Lambertian image formation model, object reflectance properties are derived independent of the spectral energy distribution of the illuminant. Gaussian spectral and spatial probes are used to estimate the proposed differential invariant. Knowledge about the spectral power distribution of the illuminant is not required for the proposed invariant.

The physical approach to color constancy offered in the paper confirms relational color constancy as a first step in color constant vision systems. Hence, low-level mechanisms as color constant edge detection reported here may play an important role in front-end vision. The research presented raises the question whether the illuminant is estimated at all in pre-attentive vision.

## Introduction

A well known property of human vision, known as color constancy, is the ability to correct for color deviations caused by a difference in illumination. Although the effect is a long standing research topic [12, 14, 18], the mechanism involved is only partly resolved.

A common approach to investigate color constant behavior is by psychophysical experiments [1, 12, 13]. Despite the exact nature of such experiments, there are intrinsic difficulties to explain the experimental results. For relatively simple experiments, the results may not explain in enough detail the mechanism underlying color constancy. For example, in [13] the same stimulus patch, either illuminated by the test illuminant, or by the reference illuminant, was presented to the left and right eye. The subject was asked to match the appearance of the color under the reference illuminant to the color under the test illuminant.

As discussed by the authors, the experiment is synthetical in that the visual scene lacks a third dimension. Although the results correspond to their predictions, they are unable to prove their theory on natural scenes, the scenes where shadow plays an important role. On the other hand, for complex experiments, with inherently a large amount of variables involved, the results does not describe color constancy isolated from other perceptual mechanisms. In [1], a more natural scene is used, in that objects were placed in the experimentation room. The observer judged the appearance of a test patch mounted on the far wall of the room. The observer was asked to vary the chromaticity of the test patch so that it appeared achromatic. The color constancy reported is excellent, but the experiments could not be interpreted in enough detail to explain the results. Hence, a fundamental problem in experimental colorimetry is that the complex experimental environment necessary to examine color constancy makes it hard to draw conclusions.

An alternative approach to reveal the mechanisms involved in color constancy is by considering the spectral image formation. Modeling the physical process of spectral image formation provides insight into the effect of different parameters on object reflectance [3, 4, 5, 8, 17, 2]. In this paper, we aim at a physical basis for color constancy rather than a psychophysical one.

When considering the estimation of material properties on the basis of local measurements, differential equations constitute a natural framework to describe the physical process of image formation. A well known technique from scale-space theory [11] is the convolution of a signal with a derivative of the Gaussian kernel to obtain the derivative of the signal. The Gaussian function regularizes the underlying distribution, resulting in robustness against noise. The standard deviation  $\sigma$  of the Gaussian determines the observation scale. Introduction of wavelength in the scale-space paradigm, as suggested by Koenderink [10], leads to a spatio-spectral family of Gaussian aperture functions. These color receptive fields are in [7] as the Gaussian color model. The Gaussian color model provides a physical basis, which is compatible with colorimetry, for the measurement of color constant object properties.

The color constancy problem is often posed as retrieving the unknown illuminant from a given scene [3, 13, 17,

2]. Different from their approach, features invariant to a change in illuminant can be developed [4, 5, 8]. In this paper, we focus on differential expressions which are robust to a change in illumination color. Additionally, robustness against changes in the imaging conditions, such as camera viewpoint, illumination direction, and object geometry is achieved.

The organization of the paper is as follows. Section 2 derives illumination invariant differential expressions. Measurement of spatio-spectral differential quotients is described in Section 3. Finally, a confrontation between physics based and perception based color constancy is given in Section 4.

## Illumination Invariant Properties of Object Reflectance

Any method for finding invariant color properties relies on a photometric model and on assumptions about the physical variables involved. For example, hue is known to be insensitive to surface orientation, illumination direction, intensity and highlights, under a white illumination [8]. Normalized  $rgb$  is an object property for matte, dull surfaces illuminated by white light. When the illumination color varies or is not white, other object properties which are related to constant physical parameters should be measured. In this section, expressions for determining material changes in images will be derived, robust to a change in illumination color over time.

Consider the Lambertian photometric reflection model and an illumination with locally constant color,

$$E(\lambda, \vec{x}) = e(\lambda)i(\vec{x})m(\lambda, \vec{x}) \quad (1)$$

where  $e(\lambda)$  represents the illumination spectrum,  $i(\vec{x})$  the effect of shadow and shading, and  $m(\lambda, \vec{x})$  the reflectance function of the object. The assumption of locally constant color allows for the extraction of expressions describing material changes independent of the illumination. Without loss of generality, we restrict ourselves to the one dimensional case; two dimensional expressions may be derived according to [6]. Differentiation of Eq. (1) with respect to  $\lambda$  results in

$$\frac{\partial E}{\partial \lambda} = i(x)m(\lambda, x)\frac{\partial e}{\partial \lambda} + i(x)e(\lambda)\frac{\partial m}{\partial \lambda}. \quad (2)$$

Dividing Eq. (2) by Eq. (1) gives the relative differential,

$$\frac{1}{E(\lambda, x)}\frac{\partial E}{\partial \lambda} = \frac{1}{e(\lambda)}\frac{\partial e}{\partial \lambda} + \frac{1}{m(\lambda, x)}\frac{\partial m}{\partial \lambda}. \quad (3)$$

The result consists of two terms, the former depending on the illumination color and the latter depending on material properties. Since the illumination color is constant with respect to  $x$ , differentiation to  $x$  yields a material property only,

$$\frac{\partial}{\partial x} \left\{ \frac{1}{E(\lambda, x)}\frac{\partial E}{\partial \lambda} \right\} = \frac{\partial}{\partial x} \left\{ \frac{1}{m(\lambda, x)}\frac{\partial m}{\partial \lambda} \right\}. \quad (4)$$

Assuming matte, dull surfaces, and assuming a single light source,  $N_{\lambda x}$  determines changes in object reflectance,

$$N_{\lambda x} = \frac{1}{E(\lambda, x)}\frac{\partial^2 E}{\partial \lambda \partial x} - \frac{1}{E(\lambda, x)^2}\frac{\partial E}{\partial \lambda}\frac{\partial E}{\partial x} \quad (5)$$

which determines material changes independent of the viewpoint, surface orientation, illumination direction, illumination intensity and illumination color. The expression results from differentiation of Eq. (4).

The expression given by Eq. (5) is the fundamental lowest order illumination invariant. Any spatio-spectral derivative of Eq. (5) inherently depends on the body reflectance only. According to [16], a complete and irreducible set of differential invariants is obtained by taking all higher order derivatives of the fundamental invariant,

$$N_{\lambda x \lambda^m x^n} = \frac{\partial^{m+n}}{\partial \lambda^m \partial x^n} \left\{ \frac{1}{E(\lambda, x)}\frac{\partial^2 E}{\partial \lambda \partial x} - \frac{1}{E(\lambda, x)^2}\frac{\partial E}{\partial \lambda}\frac{\partial E}{\partial x} \right\} \quad (6)$$

for  $m \geq 0, n \geq 0$ .

Application of the chain rule for differentiation yields the higher order expressions in terms of the spatio-spectral energy distribution. For instance, the spectral derivative of  $N_{\lambda x}$  is given by

$$N_{\lambda \lambda x} = \frac{E_{\lambda \lambda x} E^2 - E_{\lambda \lambda} E_x E - 2E_{\lambda x} E_{\lambda} E + 2E_{\lambda}^2 E_x}{E^3} \quad (7)$$

where  $E(\lambda, x)$  is written as  $E$  for simplicity and indices denote differentiation. Note that these expressions are valid everywhere  $E(\lambda, x) > 0$ . These invariants may be interpreted as the spatial derivative of the normalized spectral slope  $N_\lambda$  and curvature  $N_{\lambda \lambda}$  of the reflectance function  $R_\infty$ . Expressions for higher order derivatives are straightforward.

Summarizing, we have derived a complete set of color constant expressions determining object reflectance. The expressions are invariant for a change of illumination over time. The major assumption underlying the proposed invariants is a single colored illumination, effectuating a spatially constant illumination spectrum. For an illumination color varying slowly over the scene with respect to the spatial variation of the object reflectance, simultaneous color constancy is achieved by the proposed invariant.

We have proven that spatial differentiation is necessary to achieve color constancy when pre-knowledge about the illuminant is not available. Hence, any color constant system should perform both spectral as well as spatial comparison in order to be invariant against illumination changes, which confirms the theory of relational color constancy as proposed in [4]. In the next section we will present how to make such spatial and spectral comparisons on a well-defined physical basis.

## Measurement of Spatio-spectral Energy

Up to this point we did establish invariant expressions describing material changes robust to a change in illumination color. These are formal expressions, exploring the infinite dimensional Hilbert space of spectra at an infinitesimal spatial neighborhood. The spatio-spectral energy distribution is only measurable at a certain spatial resolution and spectral bandwidth, yielding a limited amount of measurements. Hence, physical realizable measurements inherently imply integration over the spectral and spatial dimensions. General aperture functions, or Gaussians and its derivatives, may be used to probe the spatio-spectral energy distribution. In this section, we introduce the Gaussian color model as a general model for the measurement of spatio-spectral differential quotients. We emphasize that no essentially new color model is proposed here, but rather a theory of color measurement.

We follow [7] for introducing the Gaussian color model. Let  $E(\lambda)$  be the energy distribution of the incident light, where  $\lambda$  denotes wavelength. The spectral energy distribution may be approximated by a Taylor expansion at  $\lambda_0$ ,

$$E(\lambda) = E^{\lambda_0} + \lambda E_{\lambda}^{\lambda_0} + \frac{1}{2} \lambda^2 E_{\lambda\lambda}^{\lambda_0} + \dots . \quad (8)$$

Measurement of the spectral energy distribution with a Gaussian aperture yields a weighted integration over the spectrum. Let  $G(\lambda_0; \sigma_\lambda)$  be the Gaussian at spectral scale  $\sigma_\lambda$  positioned at  $\lambda_0$ . The observed energy in the Gaussian color model, at infinitely small spatial resolution, approaches in second order to [7, 10]

$$\hat{E}^{\sigma_\lambda}(\lambda) = \hat{E}^{\lambda_0, \sigma_\lambda} + \lambda \hat{E}_{\lambda}^{\lambda_0, \sigma_\lambda} + \frac{1}{2} \lambda^2 \hat{E}_{\lambda\lambda}^{\lambda_0, \sigma_\lambda} + \dots \quad (9)$$

where

$$\hat{E}^{\lambda_0, \sigma_\lambda} = \int E(\lambda) G(\lambda; \lambda_0, \sigma_\lambda) d\lambda \quad (10)$$

denotes the spectral intensity,

$$\hat{E}_{\lambda}^{\lambda_0, \sigma_\lambda} = \int E(\lambda) G_{\lambda}(\lambda; \lambda_0, \sigma_\lambda) d\lambda \quad (11)$$

measures the first order spectral derivative, and

$$\hat{E}_{\lambda\lambda}^{\lambda_0, \sigma_\lambda} = \int E(\lambda) G_{\lambda\lambda}(\lambda; \lambda_0, \sigma_\lambda) d\lambda \quad (12)$$

measures the second order spectral derivative. Further,  $G_\lambda$  and  $G_{\lambda\lambda}$  denote derivatives of the Gaussian with respect to  $\lambda$ . Note that, throughout the paper, we assume scale normalized Gaussian derivatives to probe the spectral energy distribution. Hence, the Gaussian color model measures the scale-normalized coefficients  $\hat{E}^{\lambda_0, \sigma_\lambda}$ ,

$\hat{E}_\lambda^{\lambda_0, \sigma_\lambda}$  and  $\hat{E}_{\lambda\lambda}^{\lambda_0, \sigma_\lambda}$  of the Taylor expansion of the Gaussian weighted spectral energy distribution at  $\lambda_0$ .

Introduction of spatial extent in the Gaussian color model yields a local Taylor expansion at wavelength  $\lambda_0$  and position  $\vec{x}_0$ . Each measurement of a spatio-spectral energy distribution has a spatial as well as spectral resolution. The measurement is obtained by probing an energy density volume in a three-dimensional spatio-spectral space, where the size of the probe is determined by the observation scale  $\sigma_\lambda$  and  $\sigma_x$ ,

$$\hat{E}(\lambda, \vec{x}) = \hat{E} + \left( \frac{\vec{x}}{\lambda} \right)^T \begin{bmatrix} \hat{E}_{\vec{x}} \\ \hat{E}_\lambda \end{bmatrix} + \frac{1}{2} \left( \frac{\vec{x}}{\lambda} \right)^T \begin{bmatrix} \hat{E}_{\vec{x}\vec{x}} & \hat{E}_{\vec{x}\lambda} \\ \hat{E}_{\lambda\vec{x}} & \hat{E}_{\lambda\lambda} \end{bmatrix} \left( \frac{\vec{x}}{\lambda} \right) + \dots \quad (13)$$

where

$$\hat{E}_{\vec{x}; \lambda^j}(\lambda, \vec{x}) = E(\lambda, \vec{x}) * G_{\vec{x}; \lambda^j}(\lambda, \vec{x}; \sigma_\lambda, \sigma_x) . \quad (14)$$

Here,  $G_{\vec{x}; \lambda^j}(\lambda, \vec{x}; \sigma_\lambda, \sigma_x)$  are the spatio-spectral probes, or color receptive fields. The coefficients of the Taylor expansion of  $\hat{E}(\lambda, \vec{x})$  represent the local image structure completely. Truncation of the Taylor expansion results in an approximate representation, which is best possible in the least squares sense.

For human vision, the Taylor expansion is spectrally truncated at second order [9]. Hence, higher order derivatives do not affect color as observed by the human visual system. The Gaussian color model approximates the Hering basis for human color vision when taking the parameters  $\lambda_0 \simeq 520 \text{ nm}$  and  $\sigma_\lambda \simeq 55 \text{ nm}$  [7]. For this case, the measured differential quotients are denoted by  $\hat{E}$ ,  $\hat{E}_\lambda$  and  $\hat{E}_{\lambda\lambda}$ , taking the spectral scale  $\sigma_\lambda$  and position  $\lambda_0$  for granted. The spectral measurements may be interpreted as measuring intensity ( $\hat{E}$ ), yellow-bluish ( $\hat{E}_\lambda$ ), and red-greenish ( $\hat{E}_{\lambda\lambda}$ ).

It may be concluded from [7] that measurement of spatio-spectral energy implies probing the energy distribution with Gaussian apertures at a given observation scale. Probing the spectral energy density with Gaussian derivative apertures result in the decomposition of the spectrum in its Taylor expansion. The human visual system measures the intensity, slope and curvature of the spectral energy distribution, at fixed  $\lambda_0$  and fixed  $\sigma_\lambda$ . Hence, the spectral intensity and its first and second order derivatives only, combined in the spatial derivatives up to a given order, describe the local structure of a color image.

## Discussion

This paper presents a physics-based background for color constancy, valid for Lambertian light reflectance. By considering spatial and spectral derivatives of the image formation model, object reflectance properties are derived independent of the spectral energy distribution of the illuminant. Knowledge about the spectral power distribution of the illuminant is not required for the proposed invariant, as opposed to the well known von Kries transform for color constancy [18].

The robustness of our invariant Eq. (5) is assured by using the Gaussian color model, introduced in [7]. The Gaussian color model is considered an adequate approximation of the human tri-stimulus sensitivities. The Gaussian color model measures the intensity, first, and second order derivative of the spectral energy distribution, combined in a well-established spatial observation theory. Application of the Gaussian color model in color constancy ensures compatibility with colorimetry, while inherently physically sound and robust measurements are derived.

From a different perspective, color constancy was considered in [13, 1]. The background is experimental colorimetry, where subjects are asked to match the reference and test illumination condition. As a consequence their experiments do not include shadow and shading. The result of their approach shows approximate color constancy under natural illuminants. However, their approach is unable to cope with color constancy of three dimensional scenes, where shadow plays an important role. The advantage of our physical approach over an empirical colorimetric approach, is that invariant properties are deduced from the image formation model. Our proposed Eq. (5) is designed to be insensitive to intensity changes due to the scene geometry.

There are many circumstances where explicit knowledge of illuminant is missing, especially in image retrieval from large databases, or when calibration is not practically feasible as is frequently the case in light microscopy. The proposed method requires only general knowledge about the material only, hence is applicable under a larger set of imaging circumstances.

As pointed out in [13], mechanisms responding to cone-specific contrast offer a better correspondence with human vision than by a system that estimates illuminant and reflectance spectra. The research presented here raises the question whether the illuminant is estimated at all in pre-attentive vision. The physical model presented demands spatial comparison in order to achieve color constancy, thereby confirming relational color constancy as a first step in color constant vision [4, 15]. Hence, low-level mechanisms as color constant edge detection reported here may play a role in front-end vision.

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## Biography

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