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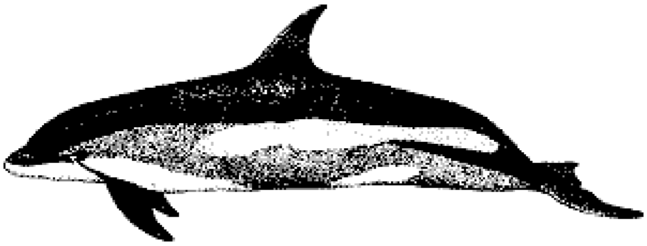

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Report P0001 januari 2000

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Basic Multi-Competence Programming

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Version 2.0

Abstract. BMCP (Basic Multi-Competence Programming) is a paradigm for teamwork for simple programming tasks. BMCP is simplified in the sense that programming tasks require no more than implementing a given functional specification. BMCP uses CVN (Conditional Value Notation) as a format for its functional specifications. BMCP displays seven competences, this collection being derived from an underlying family of four so-called program assessment perspectives. BMCP requires fixed agreements on interfacing, input and output. This version (2.0) replaces the earlier version 1.0 of this document.

Contents

1 Introduction

Multi-Competence Programming (MCP) is a programming framework meant for larger teams. Its objective is to get a firm grip on quality control supplementary to, and as much as possible simultaneously with, program construction. MCP is primarily an educational framework, delineated along 17 fresh acronyms. Taking the program algebra (see [5]) definition of what a program is as a point of departure, four *program assessment perspectives* emerge, each of which gives rise to a quality control *competence*. In terms of program algebra, a program is viewed as a text. The algorithmic content of this text is to be found via a translation (projection) to a text in a low level language (e.g., PGLA from [5]). The latter text can be viewed as a sequence of elementary instructions.

According to this view a program is a description of a sequence of instructions, the description perhaps given in a high level language which must be projected down to a low level notation in order to determine its (algorithmic) meaning. This view mainly matters if semantic issues are at stake. When programming in a known program notation, all semantic issues having been resolved already, the program algebra view must be complemented with other views of a pragmatic nature. Four of these views, or program assessment perspectives, are

the *black box test* perspective,
the *white box test* perspective,
the *white box review* perspective, and
the *white box verification* perspective.

Each of these perspectives gives rise to a competence. The competence refers to the (proven, accredited) ability to assess software from that perspective. It also refers to a role that a person or team can play in a software development activity. We get then the following acronyms, referring to the perspectives mentioned above:

BBTC:	Black Box Test Competence
WBTC:	White Box Test Competence
WBRC:	White Box Review Competence
WBVC:	White Box Verification Competence

These four perspectives and competences are further discussed in Sections 2 and 3. In a program development project based on MCP each of these competences is put in the hands of a different team. Because these competences are so totally different in practice, each having its own expert community, journals, guru's etc., we use the phrase multi-competence programming for a project planning involving all or most of these.

Having established the quality control related competences, three more competences are needed to keep a project up and running:

PCC:	Program Construction Competence
PMC:	Project Management Competence
KMC:	Knowledge Management Competence

Altogether seven competences emerge. The teams responsible for these competences are denoted with:

PCT:	Program Construction Team
PMT:	Program Management Team
KMT:	Knowledge Management Team
BBTT:	Black Box Test Team
WBTT:	White Box Test Team
WBRT:	White Box Review Team
WBVT:	White Box Verification Team

The MCP method has immediate implications for a class room setting. When a group of students is constructing programs in order to develop and/or demonstrate their programming abilities, the assessment of this output requires some care. In fact each student ought to be role aware. When produced programs are judged it must be known:

- from what perspective(s) this judgement will be made;
- how the persons responsible for making the judgement will approach their task.

MCP in its simplest form poses a programming problem in terms of a functional specification that has to be implemented. The program construction activity produces *proposed implementations*. Only the management role can upgrade a proposed implementation to the level of an implementation. This will always require some form of quality control. All major perspectives on product oriented (not: process oriented) quality control are represented in the scheme of four perspectives discussed above.

MCP uses CVN (Conditional Value Notation) as its format for functional specifications. This notation is introduced in Section 4. The advantages of CVN are:

- very concise and compact (though limited in its usability),
- captures the main error conditions: abrupt program abortion (leading to M, the meaningless state) and program divergence (leading to D, the divergent state),
- it uses conditions and quantifiers in a form which easily deals with D and M as mentioned above.

When programming tasks are simplified to the implementation of a given functional specification, avoiding any feedback to the specification itself, the framework is termed BMCP, Basic MCP. Otherwise it is called AMCP for Adaptive MCP. AMCP views the production of the functional specification as a competence as well. The specification production goes through various repetitions, including a range of program construction and assessment activities, which in turn lead to a next phase of the specification. This involves yet another competence, the *requirements analysis competence*. AMCP can only take place in a significantly more involved setting where the programs are used to support in reaching further objectives.

BMCP is considered a useful "educational" approximation of AMCP, even if BMCP is not so practical because AMCP happens to occur more in industrial projects. As an educational paradigm we first concentrate on BMCP and only thereafter consider projects involving AMCP. BMCP contains the bare essentials of programming and of programming related quality control.

There is a huge literature on software management describing numerous tools, techniques, methodologies and standards. We mention [1], [7], [9], [10], [13] and [18].

2 Four Perspectives on a Program

The description of 'what a program is' in program algebra [5] by no means exhausts the valid perspectives one may have on programs. Accepting the viewpoint that a program is a text which is given algorithmic content by a projection into a low level program notation, still four different perspectives on a program can be determined:

Black box (behavioral) perspective. A program is a black box (e.g., a compiled binary, not a "Black box", which is orange). All one can do is to run it and to perform black box tests to assess its behavior. (See e.g., [17].)

White box (behavioral) perspective. Program P is a known text. Properties of P are taken from its specification. These properties are assessed by means of black box tests and white box tests. White box tests take advantage of knowledge of the program text in order to spot difficulties more efficiently than in black box (random) mode.

White box literary perspective. The program text is to be assessed by means of informal but systematic judgement of the text (code walkthrough etc., see e.g., [12]).

White box verification perspective. The program is a text characterized by properties which have to be demonstrated in a rigorous (formal) fashion. Preferably, proofs are machine readable and machine checked. (Cf. [2].)

Each of these has its value in practice. Some implications of these perspectives are the following activities, mainly used for quality control:

Black box testing. Black box tests are generated from the specification (possibly automatically). A test environment is produced in which many tests can be run (automatically) on any proposed implementation of the specification. It is essential that proposed implementations strictly adhere to boundary conditions regarding interfaces.

White box testing. White box test can often also be generated automatically. It is important that adequate coverage is achieved, i.e., as much as possible all branches of the program must be visited by at least one test run.

Program verification. Proof generation and proof checking (if possible computer supported or automatically).

White box critical review. Here the program is understood as a pseudocode containing the solution of a problem (i.e., how to best implement the specification). In this perspective the fundamental merits of the proposed solution are considered. Speed of calculation, memory usage, clarity, independence of problematic language features, exploitation of interesting language/machine options etc.

3 Competence partitioning in BMC

In a BMC project seven different roles are to be distinguished. These roles are performed by a team. A team may consist of just one person, it is even possible that a person performs the work of different teams at the same time. The following teams can exist:

KMT (the knowledge management team) investigates the following questions:

- What kind of algorithm can be used; what can be expected/achieved?
- Which program constructions will be needed?
- What kind of verification techniques can used? how difficult will this be?
- How can BB/WB tests be derived in the case at hand?

KMT will not only produce some rationale for a proposed implementation but also a context in which the level of ambition of the proposal can be assessed.

PCT (the program construction team) generates a program and applies modifications if the interaction with any of the other teams makes that plausible.

BBTT (the black box test team) produces test beds for proposed implementations (given the specification of the required functionality).

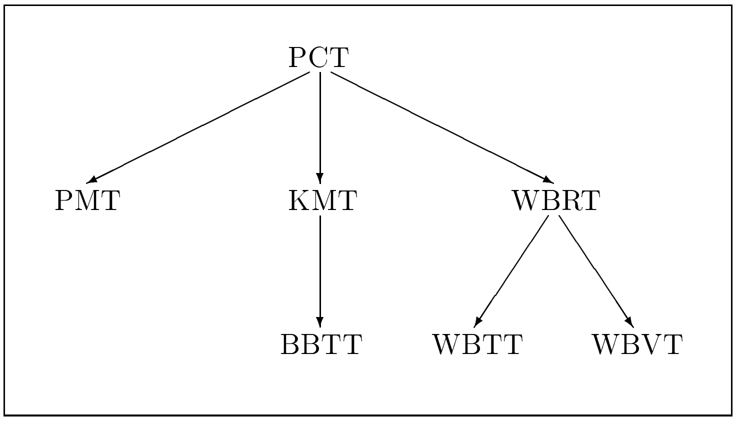
WBTT (the white box test team) delivers white box tests, preferably also in a largely automated setting.

WBRT (the white box review team) produces assessment of proposed implementations based on code inspections of various kinds.

WBVT (the white box verification team) produces correctness proofs.

PMT (the project management team) surveys the entire operation and manufactures the final deliverable containing programs as well as results of BBTT, WBTT, WBRT, WBVT, and KMT.

Amongst these teams, we discern the following hierarchy:



As suggested above, PCT takes the lead. PCT members intend to own the software. PCT uses KMT to produce background knowledge. KMT can manage BBTT as this requires specification expertise only. PCT uses WBRT for permanent assessment. WBRT in turn will guide both WBTT and WBVT (using insights of KMT). PCT uses the service of PMT to obtain better overall performance. For this reason PMT needs some power. This power is handed over by PCT in order to optimize PMT's role.

4 Conditional Value Notation

CVN, Conditional Value Notation, is a simple set of conventions to denote data values in the presence of possible errors. For each data type (data domain) V the presence of two exceptional values is assumed:

> D, a divergent value (the value of a diverging computation);
>
> M, a meaningless value (the value of an aborting computation).

Data values different from these exceptional values are called *good*. The predicate $g(\_)$ determines the good objects, i.e., $g(X)$ "holds" for some X in V if X is not D or M.[1] Thus

$$V = V_g \cup \{D, M\}$$

where $V_g = \{X \in V \mid g(X) \text{ holds}\}$. On V the symbol $=$ denotes equality, and $\neq$ denotes non-equality. With the special exception values D and M it often is simpler to use an asymmetric type of equality, called *left sequential equality* with notation $\mathrel{\underset{\circ}{=}}$. The idea is that $X \mathrel{\underset{\circ}{=}} Y$ gets its meaning by interpretation from left to right. For example, $M \mathrel{\underset{\circ}{=}} X$ equals M because equating the meaningless value with anything else is "meaningless". On $V = V_g \cup \{D, M\}$ left sequential equality is defined by:

$$\boxed{\begin{array}{lcll}
D \mathrel{\underset{\circ}{=}} X & = & D & \\
M \mathrel{\underset{\circ}{=}} X & = & M & \\
X \mathrel{\underset{\circ}{=}} D & = & D & \text{if } g(X) \text{ holds} \\
X \mathrel{\underset{\circ}{=}} M & = & M & \text{if } g(X) \text{ holds} \\
X \mathrel{\underset{\circ}{=}} X & = & T & \text{if } g(X) \text{ holds} \\
X \mathrel{\underset{\circ}{=}} Y & = & F & \text{if } g(X) \text{ and } g(Y) \text{ hold, and } X \neq Y
\end{array}}$$

[1] Instead of regarding a predicate as determining a subset, one might consider it as a function to $\{T, F\}$ and set $g(X) = T$ is $g(X)$ holds, and $g(X) = F$ otherwise.

For $X \underset{\circ}{=} Y = F$ we also write $X \underset{\circ}{\neq} Y$. Furthermore, in the case that other relations are defined over V_g, these can always be extended to V (including the error values) in a similar way. For example, in the domain $\mathbb{N}$ of the natural numbers the relation $<$ ("smaller than") can be extended to a relation $\circ\!\!<$ over $V = \mathbb{N} \cup \{D, M\}$ as follows:

$$\boxed{\begin{array}{lcll}
D \circ\!\!< X & = & D & \\
M \circ\!\!< X & = & M & \\
X \circ\!\!< D & = & D & \text{if } g(X) \text{ holds} \\
X \circ\!\!< M & = & M & \text{if } g(X) \text{ holds} \\
X \circ\!\!< Y & = & T & \text{if } g(X) \text{ and } g(Y) \text{ hold, and } X < Y \\
X \circ\!\!< Y & = & F & \text{if } g(X) \text{ and } g(Y) \text{ hold, and } X \geq Y
\end{array}}$$

It is evident how addition $(+)$ should be extended to $\circ\!\!+$ in $\mathbb{N} \cup \{D, M\}$. It is perhaps less evident how to extend subtraction $(-)$ to $\circ\!\!-$ in $\mathbb{N} \cup \{D, M\}$. Here the typical difference with $\circ\!\!+$ is caused by subtraction being a *partial* function on $\mathbb{N}$, i.e., $X - Y$ is not defined if $X < Y$. This problem is solved in $\mathbb{N} \cup \{D, M\}$ by the following equations:

$$\begin{array}{ll}
X \circ\!\!- Y = M & \text{if } g(X) \text{ and } g(Y) \text{ hold, and } X < Y \\
X \circ\!\!- Y = X - Y & \text{if } g(X) \text{ and } g(Y) \text{ hold, and } X \geq Y.
\end{array}$$

Consequently, $\circ\!\!-$ is not a partial function on $\mathbb{N} \cup \{D, M\}$.

As a convention we shall further use the common relation and operation symbols if we are sure that their arguments cannot be error values. (However, as the last example showed, the result value can be an error.)

Exercises.

1. Spell out the complete definition for subtraction on $\mathbb{N} \cup \{D, M\}$.

2. A different view on subtraction on $\mathbb{N}$ is to consider this operation as a partial one that may *diverge*. The idea is that

   $$psub(X, Y) = \textbf{if } X \underset{\circ}{=} Y \textbf{ then } 0 \textbf{ else } S(psub(X, S(Y))).$$

   (This example is taken from [3].) Here S is the *successor* function: $S(X) = X + 1$. Thus $psub(X, Y) = X - Y$ if $X \geq Y$. Describe how $psub$ can be defined on $\mathbb{N} \cup \{D, M\}$ if we set $S(D) = D$ and $S(M) = M$.

Four-Valued Conditions. In particular for the truth values T and F, instead of working with $V = \{T, F\}$ CVN proposes the use of $\mathbb{T}_4 = \{T, F, D, M\}$ (notations from [4]). We shortly elaborate on this setting, as to introduce the underlying logic over $\mathbb{T}_4$ that is useful for CVN. The logical connectives relevant for CVN are:

$\neg$	(negation),
$\mathbin{{}_{\circ}\!\wedge}$	(left sequential conjunction),
$\mathbin{{}^{\circ}\!\vee}$	(left sequential disjunction),
$\mathrel{\circ\!\!\rightarrow}$	(left sequential implication).

The (binary) connectives all have *right sequential* counterparts, e.g., $\mathbin{\wedge\!{}_{\circ}}$ is called right sequential conjunction and is defined by $X \mathbin{\wedge\!{}_{\circ}} Y = Y \mathbin{{}_{\circ}\!\wedge} X$.

So-called *truth tables* for $\neg$, $\mathbin{{}_{\circ}\!\wedge}$ and $\mathbin{{}^{\circ}\!\vee}$ are

X	$\neg X$
M	M
T	F
F	T
D	D

$\mathbin{{}_{\circ}\!\wedge}$	M	T	F	D
M	M	M	M	M
T	M	T	F	D
F	F	F	F	F
D	D	D	D	D

$\mathbin{{}^{\circ}\!\vee}$	M	T	F	D
M	M	M	M	M
T	T	T	T	T
F	M	T	F	D
D	D	D	D	D

These tables provide the definition of the logical connectives in $\mathbb{T}_4$, e.g., $M \mathbin{{}_{\circ}\!\wedge} F = M$ and $F \mathbin{{}_{\circ}\!\wedge} M = F$. Notice that $\mathbin{{}^{\circ}\!\vee}$ can be defined by *dualization*:

$$X \mathbin{{}^{\circ}\!\vee} Y = \neg(\neg X \mathbin{{}_{\circ}\!\wedge} \neg Y),$$

where the "=" is plain equality over $\mathbb{T}_4$. In this style the connective $\mathrel{\circ\!\!\rightarrow}$ is defined by $X \mathrel{\circ\!\!\rightarrow} Y = \neg X \mathbin{{}^{\circ}\!\vee} Y$. As another example of the use of this logic consider $\mathbb{N} \cup \{D, M\}$: the relation $\mathrel{{}_{\circ}\!\geq}$ can now be defined by

$$X \mathrel{{}_{\circ}\!\geq} Y = \neg(X \mathrel{\circ\!\!<} Y).$$

Exercises.

3. Determine the values of
   (a) $T \mathrel{\circ\!\!\rightarrow} (M \mathbin{{}^{\circ}\!\vee} T)$.
   (b) $(T \mathbin{{}_{\circ}\!\wedge} M) \mathbin{{}^{\circ}\!\vee} D)$.
   (c) $T \mathbin{{}_{\circ}\!\wedge} \neg(M \mathbin{{}^{\circ}\!\vee} F)$.

4. Show that the connectives $\mathbin{{}_{\circ}\!\wedge}$ and $\mathbin{{}^{\circ}\!\vee}$ are *associative*, i.e. $X \mathbin{{}_{\circ}\!\wedge} (Y \mathbin{{}_{\circ}\!\wedge} Z) = (X \mathbin{{}_{\circ}\!\wedge} Y) \mathbin{{}_{\circ}\!\wedge} Z$, and $X \mathbin{{}^{\circ}\!\vee} (Y \mathbin{{}^{\circ}\!\vee} Z) = (X \mathbin{{}^{\circ}\!\vee} Y) \mathbin{{}_{\circ}\!\wedge} Z$. (So we need not use brackets in repeated applications of each of these connectives.)

5. Let $V = V_g \cup \{D, M\}$ and consider the predicate g as a function from V to $\{T, F\}$. Let furthermore equality $(=)$ be given on V_g. Argue whether the expression

$$(\neg g(X) \mathbin{{}_{\circ}\!\wedge} X) \mathbin{{}^{\circ}\!\vee} (g(X) \mathbin{{}_{\circ}\!\wedge} \neg g(Y) \mathbin{{}_{\circ}\!\wedge} Y) \mathbin{{}^{\circ}\!\vee} (X = Y)$$

   defines left-sequential equality.

If_then and If_else. For the domain V (containing D, M) two operators are proposed by CVN:

$$p \cdot\!\!\rightarrow X \quad (p \text{ a four-valued condition, } X \in V)$$

$$X \nrightarrow Y \quad (X, Y \in V)$$

The expression $p \cdot\!\!\rightarrow X$ is pronounced "if P then X", and $X \nrightarrow Y$ is pronounced "if X else Y".

The operators $\cdot\!\!\rightarrow$ and $\nrightarrow$ are determined by the following equations:

$$\boxed{\begin{array}{rcl} T \cdot\!\!\rightarrow X & = & X \\ F \cdot\!\!\rightarrow X & = & M \\ D \cdot\!\!\rightarrow X & = & D \\ M \cdot\!\!\rightarrow X & = & M \end{array}}$$

and

$$\boxed{\begin{array}{rcl} X \nrightarrow Y & = & X \ \text{ if } g(X) \text{ holds} \\ D \nrightarrow X & = & D \\ M \nrightarrow X & = & X \end{array}}$$

So, $X \nrightarrow Y$ equals X if X is a good value or if X diverges, otherwise it equals Y. We notice that $\nrightarrow$ is *associative*, i.e.

$$X \nrightarrow (Y \nrightarrow Z) = (X \nrightarrow Y) \nrightarrow Z.$$

Therefore we can omit parentheses in repeated applications. Furthermore, to enhance readability we assume that $\mathbin{{}_{\circ}\!\wedge}$, $\mathbin{{}^{\circ}\!\vee}$ bind stronger than $\mathbin{\cdot\!\!\rightarrow}$, which in turn binds stronger than $\nrightarrow$, so

$$\neg p \mathbin{{}_{\circ}\!\wedge} q \mathbin{\cdot\!\!\rightarrow} X \nrightarrow Y = ((\neg p) \mathbin{{}_{\circ}\!\wedge} q) \mathbin{\cdot\!\!\rightarrow} X) \nrightarrow Y,$$

and the brackets in $p \mathbin{\cdot\!\!\rightarrow} (X \nrightarrow Y)$ cannot be omitted.

The following identities follow easily by considering all possible values of p and q, and will be used in the sequel:

1. $p \mathbin{\cdot\!\!\rightarrow} (X \nrightarrow Y) = p \mathbin{\cdot\!\!\rightarrow} X \nrightarrow p \mathbin{\cdot\!\!\rightarrow} Y$,
2. $p \mathbin{\cdot\!\!\rightarrow} (q \mathbin{\cdot\!\!\rightarrow} X) = p \mathbin{{}_{\circ}\!\wedge} q \mathbin{\cdot\!\!\rightarrow} X$.

With the operators $\mathbin{\cdot\!\!\rightarrow}$ and $\nrightarrow$, a two-sided conditional can be found as follows:

$$X \triangleleft p \triangleright Y = p \mathbin{\cdot\!\!\rightarrow} X \nrightarrow \neg p \mathbin{\cdot\!\!\rightarrow} Y.$$

(Cf. the notation **if** p **then** X **else** Y.)

Let V be given, and let $\Phi : V_g \to \mathbb{T}_4$ be a predicate on V_g (taking errors into account). There are two *quantifiers* that we want to use in CVN:

$$\forall X \in V_g(\Phi(X)) \quad (\text{“}\Phi(X) \text{ for all } X \in V_g\text{”})$$
$$\exists X \in V_g(\Phi(X)) \quad (\text{“there exists } X \in V_g \text{ for which } \Phi(X)\text{”})$$

We describe their values as follows:

$\forall X \in V_g(\Phi(X))$: if $\Phi(X) = T$ for all $X \in V_g$, then T,
else, if $\Phi(X) = M$ for some $X \in V_g$, then M,
else, if $\Phi(X) = F$ for some $X \in V_g$, then F,
else, D.

$\exists X \in V_g(\Phi(X))$: if $\Phi(X) = M$ for some $X \in V_g$, then M,
else, if $\Phi(X) = T$ for some $X \in V_g$, then T,
else, if $\Phi(X) = D$ for some $X \in V_g$, then D,
else, F.

Notice that quantification ranges over V_g, and that

$$\forall X \in V_g(\Phi(X)) = \neg \exists X \in V_g(\neg \Phi(X)).$$

Furthermore we shall sometimes use ‘bounded’ quantification over V_g. As an example, let $V_g = \mathbb{N}$ and $i \in \mathbb{N}$, then

$$\exists j \; 0 \leq j < i(\Phi(j))$$

expresses that there is a value j such that $0 \leq j$, $j < i$ and $\Phi(j)$ holds. (This is not a real extension: use of bounded quantification can be avoided at cost of readability).

The strength of CVN is its explicit dealing with the two fundamental error cases: abrupt abortion of a computation (leading to M) and computational divergence (modeled by D). CVN can be used to specify programs. This will work in a minority of cases, but this restricted area is of importance.[2] In Sections 5 and 6 we consider two examples of program specification with CVN.

Exercises. Let $V_g = \mathbb{N}$ and let the relation $<$ (smaller than) be given.

6. Determine whether the following expressions are well-defined:

   (a) $T \mathrel{\cdot\!\!\rightarrow} 6$

   (b) $3 \mathrel{\cdot\!\!\rightarrow} 5$

   (c) $T \mathrel{\cdot\!\!\rightarrow} T$

   (d) $(3 \mathrel{{}_{\circ}{=}} 5) \mathrel{\cdot\!\!\rightarrow} 2 \nvrightarrow D$

7. Determine the *type* (data type, data domain) of the following expressions:

   (a) $T \mathrel{\cdot\!\!\rightarrow} 6$

   (b) $F \mathrel{\cdot\!\!\rightarrow} D$

   (c) $3 \mathrel{{}_{\circ}{=}} 5$

   (d) $(3 \mathrel{{}_{\circ}{=}} 5) \mathrel{\cdot\!\!\rightarrow} T$

   (e) $(3 \mathrel{{}_{\circ}{=}} 5) \mathrel{\cdot\!\!\rightarrow} 2 \nvrightarrow 7$

8. Argue whether the following holds:

$$\begin{aligned} X \propto Y \quad = \quad & g(X) \mathbin{{}_{\circ}\!\wedge} g(Y) \mathrel{\cdot\!\!\rightarrow} (X < Y) \nvrightarrow \\ & g(X) \mathbin{{}_{\circ}\!\wedge} \neg g(Y) \mathrel{\cdot\!\!\rightarrow} Y \nvrightarrow \\ & \neg g(X) \mathbin{{}_{\circ}\!\wedge} g(Y) \mathrel{\cdot\!\!\rightarrow} X \nvrightarrow \\ & \neg g(X) \mathbin{{}_{\circ}\!\wedge} \neg g(Y) \mathrel{\cdot\!\!\rightarrow} X. \end{aligned}$$

   Can you find a shorter CVN expression that defines $\propto$?

[2]Of course, many natural extensions of CVN are conceivable (e.g., with recursive or higher order ingredients). Some proposals are discussed later in the paper.

Motivation for CVN. BCMP strongly depends on the availability of a functional specification of a piece of software beforehand. The task of the BCMP project is to obtain an implementation of that specification.

Clearly such specifications must be expressed in a precise notation. There is a very rich literature about specification notations for software. Notoriously lacking, however, are simple techniques that can deal with less elegant aspects such as errors, divergencies and partiality of operators. Algebraic specification use a non-monotonic logic, but can in practice be based on monotonic term rewriting (see [6]), pseudo-algebraic specifications (present for instance in ASF + SDF [11]) use non-monotonic term rewriting instead. The logic of non-monotonic term rewriting is far from trivial unfortunately. Industrial strength methods like VDM use a three valued logic (see [16, 14]).

The notation CVN, as introduced in the present paper, can be used for the description of simple program functionalities. At the same time it deals with two kinds of errors: divergence (D) and abortion (M). CVN has been stripped from all aspects needed to make it applicable for more complex examples. No form of modularity is supported at all. This reflects the authors opinion that error handling cannot be viewed as an 'add on feature', whereas modularity does constitute an 'add on feature'. In particular the addition of error mechanisms to a complex specification notation may introduce intractable semantic questions. CVN addresses such questions right from the start. The price paid being that there is some 'overkill' in view of simple examples. Having secured a solution for error handling the development of more expressive formalisms can proceed in a systematic and uninterrupted way on top of CVN (when needed).

Two extensions of CVN. We end this section by introducing two 'natural' extensions of CVN. Let $\Phi : \mathbb{N} \to \mathbb{T}_4$ be a predicate on $\mathbb{N}$ (taking errors into account). The minimalisation operator μ ("mu") applied to Φ selects the (smallest) natural number n such that $\Phi(n) = T$ and $\forall k < n(\Phi(k) \in \{F, M\})$. If no such n exists, the minimalisation of Φ yields value D. In the latter case, minimalisation characterises either an infinite computation (for all $n \in \mathbb{N}$, $\Phi(n) \in \{F, M\}$), or one that breaks down ($\Phi(n) = D$ for certain n). As an example, let the predicate *odd* represent the odd numbers, then $\mu(odd) = 1$. A particular example is the case that $\Phi = \lambda n.M$, i.e. Φ is the predicate that yields value M for all $n \in \mathbb{N}$: now we find $\mu\Phi = D$. Taking S the *successor function* (i.e., $S(X) = X + 1$), minimalisation of Φ can be formally defined as follows:

$$\mu\Phi = \Phi(0) \cdot\!\!\to 0 \nrightarrow \mu(\lambda n.\Phi(S(n))).$$

Below we provide an example in which the minimalisation operator μ is used.

As a second extension of CVN, we introduce repeated conjunctions and disjunctions. For predicate $\Phi : \mathbb{N} \to \mathbb{T}_4$, we define:

$$
\begin{array}{ll}
{}_{\circ\circ}\!\!\bigwedge_i^k \Phi = T & \text{if } i > k \\[2ex]
{}_{\circ\circ}\!\!\bigwedge_i^k \Phi = \Phi(i) \mathbin{{}_{\circ}\!\wedge} {}_{\circ\circ}\!\!\bigwedge_{i+1}^k \Phi & \text{if } i \leq k
\end{array}
$$

and by dualization:

$$
\begin{array}{ll}
{}^{\circ\circ}\!\!\bigvee_i^k \Phi = F & \text{if } i > k \\[2ex]
{}^{\circ\circ}\!\!\bigvee_i^k \Phi = \Phi(l) \mathbin{{}^{\circ}\!\vee} {}^{\circ\circ}\!\!\bigvee_{i+1}^k \Phi & \text{if } i \leq k
\end{array}
$$

In the following example, both minimalisation and repeated conjunction occur.

Example. In the following we describe a prototypical, generic occurrence of D caused by partiality. Consider the partial subtraction function *psub* as introduced in Exercise 2, and recall our notation $p \cdot\!\!\to X \nrightarrow \neg p \cdot\!\!\to Y$ for **if** p **then** X **else** Y. So *psub* is defined by

$$
psub(X, Y) = (X \mathrel{{}_{\circ}\!\!=} Y) \cdot\!\!\to 0 \nrightarrow \neg(X \mathrel{{}_{\circ}\!\!=} Y) \cdot\!\!\to S(psub(X, S(Y))).
$$

In the case that at least one of X, Y is in $\{D, M\}$, this yields the expected value of $psub(X, Y)$. In the other case, so for $X, Y \in \mathbb{N}$, we give a further analysis of the definition of *psub*. We define $S^0(X) = X$ and $S^{k+1}(X) = S(S^k(X))$, and we further write $S(0) = 1$, $S^2(0) = 2$, etc. Unfolding the recursive definition of *psub* once, we obtain

$$
\begin{array}{rcl}
psub(X, Y) & = & (X \mathrel{{}_{\circ}\!\!=} Y) \cdot\!\!\to 0 \nrightarrow \\
& & \neg(X \mathrel{{}_{\circ}\!\!=} Y) \cdot\!\!\to [X \mathrel{{}_{\circ}\!\!=} S(Y) \cdot\!\!\to 1 \nrightarrow S^2(psub(X, S^2(Y)))] \\
& = & (X \mathrel{{}_{\circ}\!\!=} Y) \cdot\!\!\to 0 \nrightarrow \\
& & \neg(X \mathrel{{}_{\circ}\!\!=} Y) \cdot\!\!\to [X \mathrel{{}_{\circ}\!\!=} S(Y) \cdot\!\!\to 1] \nrightarrow \\
& & \neg(X \mathrel{{}_{\circ}\!\!=} Y) \cdot\!\!\to [\neg(X \mathrel{{}_{\circ}\!\!=} S(Y)) \cdot\!\!\to S^2(psub(X, S^2(Y)))] \\
& = & (X \mathrel{{}_{\circ}\!\!=} Y) \cdot\!\!\to 0 \nrightarrow \\
& & [\neg(X \mathrel{{}_{\circ}\!\!=} Y) \mathbin{{}_{\circ}\!\wedge} X \mathrel{{}_{\circ}\!\!=} S(Y)] \cdot\!\!\to 1 \nrightarrow \\
& & [\neg(X \mathrel{{}_{\circ}\!\!=} Y) \mathbin{{}_{\circ}\!\wedge} \neg(X \mathrel{{}_{\circ}\!\!=} S(Y))] \cdot\!\!\to S^2(psub(X, S^2(Y))).
\end{array}
$$

Let $\phi_i^k(X, Y) = {}_{\circ\circ}\!\!\bigwedge_i^k \neg(X \mathrel{{}_{\circ}\!\!=} S^{i-1}(Y))$ and $\psi_i^k(X, Y) = \phi_i^k(X, Y) \mathbin{{}_{\circ}\!\wedge} X \mathrel{{}_{\circ}\!\!=} S^k(Y)$.

Extending the unfolding above, we can infer for any $k > 1$ that

$$\begin{aligned} psub(X,Y) \;&=\; \psi_1^0(X,Y) \cdot\!\!\rightarrow 0 \nrightarrow \psi_1^1(X,Y) \cdot\!\!\rightarrow 1 \nrightarrow ... \nrightarrow \psi_1^k(X,Y) \cdot\!\!\rightarrow k \nrightarrow \\ &\quad\; \psi_1^{k+1}(X,Y) \cdot\!\!\rightarrow S^{k+1}(psub(X, S^{k+1}(Y))). \end{aligned}$$

It is not hard to see that $\psi_1^k(X,Y) = X \mathrel{{}_{\sigma}{=}} S^k(Y)$ for X, Y ranging over $\mathbb{N}$, so

$$psub(X,Y) = (X \mathrel{{}_{\sigma}{=}} Y) \cdot\!\!\rightarrow 0 \nrightarrow (X \mathrel{{}_{\sigma}{=}} S(Y)) \cdot\!\!\rightarrow 1 \nrightarrow (X \mathrel{{}_{\sigma}{=}} S^2(Y)) \cdot\!\!\rightarrow 2 \nrightarrow ...$$

Now set $f = \lambda n. X \mathrel{{}_{\sigma}{=}} S^n(Y)$, then it follows that

$$psub(X,Y) = \mu f.$$

As an example, $psub(1,0) = \mu f(1,0) = 1$. Furthermore, by the fact that $0 \mathrel{{}_{\sigma}{=}} S^{k+1}(0) = F$ for all $k \in \mathbb{N}$, we obtain

$$psub(0,1) = D,$$

which reflects the infinite computation resulting from the recursive definition of $psub$.

Note that μf can be taken as the *definition* of $psub$ on $\mathbb{N} \times \mathbb{N} \to \mathbb{N} \cup \{D\}$, and indeed reflects the partiality of $psub(X,Y)$ by a divergent computation when $Y > X$. Furthermore, μf also provides a correct definition for the extension of $psub$ to $\mathbb{N} \cup \{D\} \times \mathbb{N} \cup \{D\} \to \mathbb{N} \cup \{D\}$. However, on $\mathbb{N} \cup \{D, M\}$, $psub$ and μf define different operations: e.g., $psub(M,0) = M$ and $\mu f(M,0) = D$. This ends our example on the extensions of CVN.

Exercise.

8. Let $f = \mu.k \exists l, m, n((k+1)^{n+3} + (l+1)^{n+3} = (m+1)^{n+3} \nrightarrow 3)$. Argue—in a fashion similar to the argument given in the example—that $\mu f = D$. (Hint: Consult `http://www.netspace.org/herald/issues/092995/wiles.f.html`.)

5 Example A

In this section we consider the first of two examples, example **A**, of program specifications using CVN. In this and the second example we will use the notion of an *array*, i.e., a collection of indexed data. We also provide the description of a program (or better, a function) for the example and a black box tester both written in the programming language C designed by Dennis Ritchie at Bell Laboratories in about 1972 (see e.g. [15], [8]).

Input:

$max \in \mathbb{N}$,

integer array $a[0], ..., a[max]$.

We view $\{0, 1, 2, ..., max\}$ as V_g, and use notation

$[0, max]$

for this set. More precisely, $[i, j]$ represents the empty set if $i, j \in \{0, 1, 2, ..., max\}$ and $i > j$, and $\{i\} \cup [i + 1, j]$ otherwise.

Output in CVN:

$(max < 50) \cdot\!\!\rightarrow 2 \nrightarrow$

$(\forall i \in [25, max]\ (a[49] \geq a[i])) \cdot\!\!\rightarrow (a[1] + a[2]) \nrightarrow$

$(\forall i \in [0, max]\ (\forall j \in [0, max]\ (a[i] - a[j] \neq 83))) \cdot\!\!\rightarrow 12 \nrightarrow 0$

We remark that this functionality in itself is not an interesting one; it might occur in a larger collection of functionalities.

Table 1 exhibits a straightforward translation of the CVN specification into C code. The tester in Table 2 generates in `main` an array $a[0], a[1], \ldots, a[max]$ of random integers which — with the exception of $a[49]$ — are all within the interval [-`bound`,`bound`]. The length of the array, i.e., the number of integers generated, the upper and lower bound, and — in case `max` ≥ 50 — $a[49]$ are determined by the user. The produced array is written to the file `arrayA.txt`; the result computed by the function `ExampleA` is send to the standard output stream. Note that, since C integers are not arbitrary large, `bound` and $a[49]$ have to be in the range provided by the C compiler.

An exhaustive test set with `BlackboxA.c` will now contain a session similar to the one listed in Table 3. The first test yields clearly the expected result: since the generated array consists of 11 elements, the function returns 2. Inspection of the file `arrayA.txt` is in this case superfluous. Also the second test shows the right performance: the array has more than 50 elements and, since $a[49]$ coincides with the upper bound, it is an upper bound to the elements $a[25], \ldots, a[max]$. However, now we have to consult `arrayA.txt` in order to check that $-677 = a[1] + a[2]$. In the last test we generate a sufficiently long array with elements — with the exception of $a[49]$ — within the range $[-10, 10]$. $a[49]$ is assigned the value -11. $a[49]$ is therefore not an upper bound of the elements $a[25], \ldots, a[max]$, and the difference between the various elements does not exceed 21. The expected functionality is thus the return of the value 12, which is indeed the case.

```
typedef enum {false, true} bool;

int ExampleA ( int , int * );

int ExampleA ( int max , int *a )
{
  int i, j, result;
  bool cond;

  if ( max < 50 )
     result = 2;
  else {
      cond = true;
      i=25;
      while ( cond && i <= max ){
         if ( a[i] <= a[49] )
            i++;
         else
            cond = false;
      }
      if ( cond )
          result = a[1] + a[2];
      else {
          i = 0;
          cond =  true;
          while ( cond && i <= max ){
              j = 0;
              while ( cond && j <= max ){
                  if ( a[i] - a[j] != 83 )
                     j++;
                  else
                     cond = false;
              }
              i++;
          }
          if ( cond )
             result = 12;
          else
             result = 0;
      }
  }
  return result;
}
```

Table 1: `ExampleA.c`

```
#include <stdio.h>
#include <stdlib.h>
#include <time.h>
#include "ExampleA.c"

int main ( void )
{
  int i, b, max, bound, *a;
  FILE *output;

  printf( "Enter max: " );
  scanf( "%d", &max );
  printf( "Enter bound: " );
  scanf( "%d", &bound );
  srand( time(NULL) );
  output = fopen( "arrayA.txt", "w" );
  a = malloc(sizeof(i));
  for ( i=0; i<= max; i++ ){
    if ( i==49 ){
      printf( "Enter a[49]: " );
      scanf( "%d", a );
    } else {
      b = rand() % 2;
      if ( b == 0 ){
        *a = rand() % (bound + 1);
      } else {
        *a = - (rand() % (bound + 1));
      }
    }
    fprintf( output, "a[%d]=%6d\n" , i, *a );
    a++;
  }
  fclose( output );
  a = a - ( max + 1 );
  printf( "Result: %d\n", ExampleA ( max, a ) );
  return 0;
}
```

Table 2: `BlackboxA.c`

```
(bmcp@model 74) gcc -Wall -o BlackboxA BlackboxA.c
(bmcp@model 75) ./BlackboxA
Enter max: 10
Enter bound: 374
Result: 2
(bmcp@model 76) ./BlackboxA
Enter max: 460
Enter bound: 2000
Enter a[49]: 2000
Result: -677
(bmcp@model 77) more arrayA.txt
a[0]=  -927
a[1]=   928
a[2]= -1605
a[3]=   240
a[4]=  1809
.
.
.
.
(bmcp@model 78) ./BlackboxA
Enter max: 50
Enter bound: 10
Enter a[49]: -11
Result: 12
(bmcp@model 79)
```

Table 3: Typical test session for `ExampleA.c`

Exercise.

9. The test set depicted in Table 3 is not yet exhaustive, i.e., not all possible cases are taken into consideration. State the remaining case and design an appropriate test.

6 Example B

This section deals with the second example, Example B. We proceed along the same lines as in the previous section.

Input:

$$max \in \mathbb{N},$$
$$\text{integer array } a[0], ..., a[max],$$
$$\text{integer array } b[0], ..., b[max].$$

Again we set $V_g = \{0, 1, 2, ..., max\}$.

Output: We are interested in a program that will read $a[0], b[0]$, then produce $c[0]$, thereafter read $a[1], b[1]$ and produce $c[1]$ and so on till $c[max]$ has been generated. CVN is used to express $c[i]$ in $a[j], b[j]$ for $j \leq i$. The intuition is that $b[i]$ represents an instruction:

$$\begin{aligned} b[i] = 0 & \quad \text{represents insert,} \\ b[i] = 1 & \quad \text{represents delete,} \\ 0 \neq b[i] \neq 1 & \quad \text{represents test,} \end{aligned}$$

and $a[i]$ contains the value to be inserted, deleted or for which membership is tested. Initially the set which the program will maintain is empty. In CVN:

$$\begin{aligned} &((b[i] = 0) \mathbin{{}^{\circ}\!\vee} (b[i] = 1)) \cdot\!\!\rightarrow T \nrightarrow \\ &(i = 0) \cdot\!\!\rightarrow F \nrightarrow \\ &(\exists j\ 0 \leq j < i\ ((a[i] = a[j]) \mathbin{{}_{\circ}\!\wedge} (b[j] = 0) \mathbin{{}_{\circ}\!\wedge} \\ &\qquad \forall k\ j < k < i\ ((b[k] = 1) \mathbin{\circ\!\!\rightarrow} (a[k] \neq a[i])))) \cdot\!\!\rightarrow T \nrightarrow F \end{aligned}$$

Remark: again this functionality in itself is not an interesting one; it might occur in a larger collection of functionalities.

Table 4 contains the C code for Example B. The tester in Table 5 generates now in `main` two arrays $a[0], a[1], \ldots, a[max]$ and $b[0], b[1], \ldots, b[max]$ of random integers which are all within the interval [-`bounda`,`bounda`] and [0,`boundb`], respectively. The length of the arrays, the upper and lower bounds, and the index `i` of an element in the array $c[0], c[1], \ldots, c[max]$ are determined by the user. The produced arrays are written to the file `arrayB.txt`; the value of the element c[`i`] is send to the standard output stream. In Table 6 we listed a few example tests some of which require inspection of the generated arrays.

Exercise.

10. Design a complete series of tests for `ExampleB.c`

```
typedef enum {false, true} bool;
typedef struct {
  int a;
  int b;
} AB;

char ExampleB ( int , AB * );

char ExampleB ( int i , AB *ab )
{
  int j, k;
  bool cond;
  char result;

  if ( ab[i].b == 0 || ab[i].b == 1 ) {
     result = 'T';
  } else {
    if ( i == 0 ){
      result = 'F';
    } else {
      j = 0;
      cond = false;
      while ( !cond && j < i ){
        if ( ab[i].a == ab[j].a && ab[j].b == 0 ){
          k = j+1;
          cond = true;
          while ( cond && k < i ){
            if ( ab[k].b != 1 || ab[k].a != ab[i].a ){
              k++;
            } else cond = false;
          }
        }
        j++;
      }
      if ( cond ) result = 'T';
      else result = 'F';
    }
  }
  return result;
}
```

Table 4: `ExampleB.c`

```
#include <stdio.h>
#include <stdlib.h>
#include <time.h>
#include "ExampleB.c"

int main ( void )
{
  int i, n, max, bounda, boundb;
  AB  *ab;
  FILE *output;

  printf( "Enter max:" );
  scanf( "%d", &max );
  printf( "Enter bounda:" );
  scanf( "%d", &bounda );
  printf( "Enter boundb:" );
  scanf( "%d", &boundb );
  srand( time(NULL) );
  output = fopen( "arrayB.txt", "w" );
  ab = malloc(sizeof(AB));
  for ( i = 0; i <= max; i++ ){
    n = rand() % 2;
    if ( n == 0 ){
      (*ab).a = rand() % (bounda + 1);
    } else {
      (*ab).a = - rand() % (bounda + 1);
    }
    (*ab).b = rand() % (boundb + 1);
    fprintf( output, "a[%d]=%6d \t b[%d]=%6d\n", i, (*ab).a, i, (*ab).b );
    ab++;
  }
  fclose( output );
  ab = ab - (max + 1);
  printf( "Enter i: " );
  scanf( "%d", &i );
  printf( "c[%d]=%c\n", i, ExampleB(i,ab) );
  return 0;
}
```

Table 5: BlackboxB.c

```
(bmcp@model 14) gcc -Wall -o BlackboxB BlackboxB.c
(bmcp@model 15) ./BlackboxB
Enter max:23
Enter bounda:41
Enter boundb:2
Enter i: 17
c[17]=T
(bmcp@model 16) ./BlackboxB
Enter max:34
Enter bounda:957
Enter boundb:1102
Enter i: 0
c[0]=F
(bmcp@model 17) more arrayB.txt
a[0]=   401      b[0]=   745
a[1]=   946      b[1]=    23
a[2]=   664      b[2]=   387
a[3]=  -892      b[3]=   478
a[4]=  -431      b[4]=    25
.                .
.                .
.                .
(bmcp@model 18) ./BlackboxB
Enter max:181
Enter bounda:749
Enter boundb:1275
Enter i: 1
c[1]=F
(bmcp@model 19) more arrayB.txt
a[0]=  -387   b[0]=   251
a[1]=   -48   b[1]=  1252
a[2]=  -141   b[2]=   480
a[3]=  -129   b[3]=   961
a[4]=  -470   b[4]=   464
.                .
.                .
.                .
(bmcp@model 20)
```

Table 6: Sample tests for ExampleB.c

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